Wireless Network Pricing
Chapter 5: Monopoly and Price Discriminations

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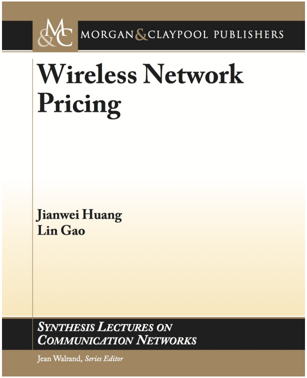
The Book



- E-Book freely downloadable from NCEL website: `http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing`
- Physical book available for purchase from Morgan & Claypool (`http://goo.gl/JFGlai`) and Amazon (`http://goo.gl/JQKaEq`)

Chapter 5: Monopoly and Price Discriminations

Focus of This Chapter

- Key Focus: This chapter focuses on the problem of profit maximization in a monopoly market, where one service provider (monopolist) dominates the market and seeks to maximize its profit.
- Theoretic Approach: Price Theory
  - Price theory mainly refers to the study of how prices are decided and how they go up and down because of economic forces such as changes in supply and demand (from Cambridge Business English Dictionary)

Price Theory

- Follow the discussions in "Price Theory and Applications" by B. Peter Pashigian (1995) and Steven E. Landsburg (2010)
- Part I: Monopoly Pricing
  - The service provider charges a single optimized price to all the consumers;
- Part II: Price Discrimination
  - The service provider charges different prices for different units of products or to different consumers.

Section 5.1
Theory: Monopoly Pricing

What is Monopoly?

- Etymology suggests that a "monopoly" is a single seller, i.e., the only firm in its industry.
  - Question: Is Apple a monopoly?
    - ★ It is the only firm that sells iPhone;
    - ★ It is *not* the only firm that sells smartphones.
- The formal definition of monopoly is based on the monopoly power.

Definition (Monopoly)

A firm with monopoly power is referred to as a monopoly or monopolist.

What is Monopoly Power?

- Monopoly power (or market power) is the ability of a firm to affect market prices through its actions.

Definition (Monopoly Power)

A firm has monopoly power, if and only if

(i) it faces a downward-sloping demand curve for its product, and

(ii) it has no supply curve.

  - (i) implies that a monopolist is not perfectly competitive. That is, he is able to set the market price so as to shape the demand.
  - (ii) implies that the market price is a consequence of the monopolist's actions, rather than a condition that he must react to.

Profit Maximization Problem

- P: the market price that a monopolist chooses;
- $Q \triangleq D(P)$: the downward-sloping demand curve that the monopolist faces;

Definition (Monopolist's Profit Maximization Problem)

How should the monopolist choose the market price P to maximize his profit $\pi(P)$, where

$$\pi(P) \triangleq P \cdot Q = P \cdot D(P).$$

Profit Maximization Problem

- The first-order condition:

$$\frac{\mathrm{d}\pi(P)}{\mathrm{d}P} = Q + P \cdot \frac{\mathrm{d}Q}{\mathrm{d}P} = 0$$

- The optimality condition:

$$\frac{P \cdot \triangle Q}{Q \cdot \triangle P} + 1 = 0$$

  - $\triangle P$ is a very small change in price, and $\triangle Q$ is the corresponding change in demand quantity.

Demand Elasticity

- Price Elasticity of Demand (defined in Section 3.2.5)

$$\eta \triangleq \frac{\triangle Q/Q}{\triangle P/P} = \frac{P \cdot \triangle Q}{Q \cdot \triangle P}$$

  - The ratio between the percentage change of demand and the percentage change of price.

- A Closely Related Question: *Under a particular price P and demand $Q = D(P)$, how much does the monopolist have to lower his price to sell additional $\triangle Q$ units of product?*

$\Rightarrow$ Answer:

$$\triangle P = \frac{P}{Q \cdot \eta} \cdot \triangle Q$$

Demand Elasticity

- The monopolist's total profit changes by selling additional $\triangle Q$ units of product:

$$\begin{aligned}\triangle\pi &\triangleq P\cdot\triangle Q-|\triangle P|\cdot Q\\ &=P\cdot\triangle Q-\left|\frac{P}{Q\cdot\eta}\cdot\triangle Q\right|\cdot Q\\ &=P\cdot\triangle Q\cdot\left(1-\frac{1}{|\eta|}\right)\end{aligned}$$

  - $P\cdot\triangle Q$ is the profit gain that the monopolist achieves, by selling additional $\triangle Q$ units of product at price P;
  - $|\triangle P|\cdot Q$ is the profit loss that the monopolist suffers, due to the decrease of price (by $|\triangle P|$) for the previous Q units of product.

Demand Elasticity

- Monopolist's Total Profit Change:

$$\triangle\pi = P \cdot \triangle Q \cdot \left(1 - \frac{1}{|\eta|}\right)$$

  - If $|\eta| > 1$, then $\triangle\pi > 0$. This implies that the monopolist has incentive to decrease the price when $|\eta| > 1$.
  - If $|\eta| < 1$, then $\triangle\pi < 0$. This implies that the monopolist has incentive to increase the price when $|\eta| < 1$.
  - If $|\eta| = 1$, then $\triangle\pi = 0$. This implies that the monopolist has no incentive to increase or decrease the price when $|\eta| = 1$ (assuming no producing cost).

- The price under $|\eta| = 1$ is the optimal price (if no producing cost)
  - Equivalent to the previous first-order condition.

Demand Elasticity

- Suppose the unit producing cost is C. Then, the optimal price is given by $\triangle\pi = \triangle C \triangleq C \cdot \triangle Q$, or equivalently,

$$P \cdot \triangle Q \cdot \left(1 - \frac{1}{|\eta|}\right) = C \cdot \triangle Q.$$

- Hence at the optimal price, we have

$$|\eta| = \frac{1}{1 - C/P} > 1$$

- Recall that
  - When $|\eta| > 1$, we say that the demand curve is elastic.
  - When $|\eta| < 1$, we say that the demand curve is inelastic.

Theorem

A monopolist always operates on the elastic portion of the demand curve.

Demand Elasticity

- When $\triangle Q = 1$, then
  - $\triangle \pi = P \cdot \left(1 - \frac{1}{|\eta|}\right)$ is usually called the marginal revenue (MR);
  - $\triangle C$ is usually called the marginal cost (MC);
- Hence the optimal monopoly price equalizes the marginal revenue and marginal cost,

$$\triangle \pi = \triangle C.$$

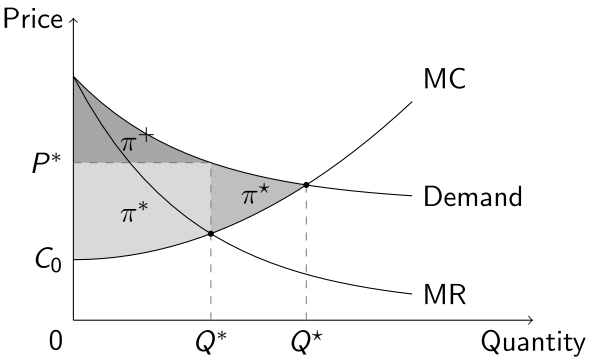
Section 5.2
Theory: Price Discriminations

What is Price Discrimination?

- Price discrimination (or price differentiation) is a pricing strategy where products are transacted at different prices in different markets or territories.
- Examples of Price Discriminations:
  - Charge different prices to the same consumer, e.g., for different units of products;
  - Charge uniform but different prices to different groups of consumers for the same product.
- Types of Price Discrimination
  - First-degree price discrimination
  - Second-degree price discrimination
  - Third-degree price discrimination

An Illustrative Example

- Example: *How the monopolist increase his profit via price discrimination?*
  - MR: the marginal revenue (profit) curve;
  - MC: the marginal cost curve;
  - Demand: the downward-sloping demand curve;



Without Price Discrimination

- Without price discrimination, the monopolist charges a single monopoly price to all consumers:
- The optimal monopoly price is P^* and the demand is Q^* (intersection of MC and MR curves);
- The monopolist's profit is π^*, and the consumer surplus is π^+.

With Price Discrimination

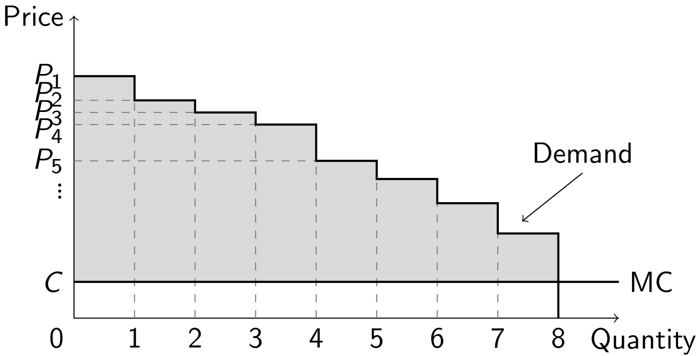
- With price discrimination, the monopolist can charge different prices to different consumers:
- For example, the monopolist can charge each consumer the most that he would be willing to pay for each product that he buys;
- With the same demand Q^*, the monopolist's profit is $\pi^* + \pi^+$, and the consumer surplus is 0;
- When the demand increases to $Q^\star$, the monopolist's profit is $\pi^* + \pi^+ + \pi^\star$, and the consumer surplus is 0;

First Degree Price Discrimination

- With the **first-degree price discrimination** (or perfect price discrimination), the monopolist charges each consumer the most that he would be willing to pay for each product that he buys.
- The monopolist captures all the market surplus, and the consumer gets zero surplus.
- It requires that the monopolist knows exactly the maximum price that every consumer is willing to pay for each product, i.e., the full knowledge about every consumer demand curve.

Illustration of First Degree Price Discrimination

- The consumer is willing to pay a maximum price P_1 for the first product, P_2 for the second product, and so on.
- Under the first-degree price discrimination, the consumer is charged by P_1 for the first product, P_2 for the second product, and so on.
- The monopolist captures all the market surplus (shadow area).



Second Degree Price Discrimination

- With the **second-degree price discrimination** (or declining block pricing), the monopolist offers a bundle of prices to each consumer, with different prices for different blocks of units.
- The second-degree price discrimination can be viewed as a limited version of the first-degree price discrimination (where a different price is set for every different unit).
- The second-degree price discrimination can be viewed as a generalized version of the monopoly pricing (as it degrades to the monopoly pricing when the number of prices is one).

Illustration of Second Degree Price Discrimination

- Under this second-degree price discrimination, the monopolist offers a bundle of prices $\{P_1, P^*, P_2\}$ with $P_1 > P^* > P_2$.
  - P_1 is the unit price for the first block (the first Q_1 units) of products;
  - P^* is the unit price for the second block (from Q_1 to Q^*) of products;
  - P_2 is the unit price for the third block (from Q^* to Q_2).
  - The monopolist's profit is illustrated by the shadow area, and the consumer surplus is $\delta_1 + \delta^* + \delta_2$.

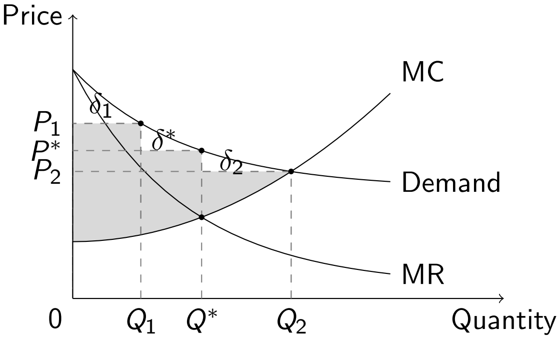


Illustration of Second Degree Price Discrimination

- Under the second-degree price discrimination $\{P_1, P^*, P_2\}$:
  - The monopolist's profit is illustrated by the shadow area, and the consumer surplus is $\delta_1 + \delta^* + \delta_2$.
- Under the first-degree price discrimination:
  - The monopolist charges a different price $D(Q)$ for each unit of product;
  - The monopolist captures all the market surplus (the shadow area + $\delta_1 + \delta^* + \delta_2$, and the consumer achieves azero surplus.

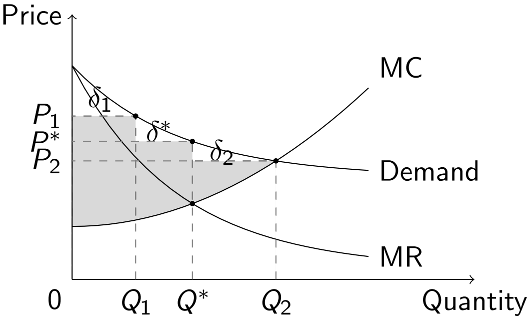
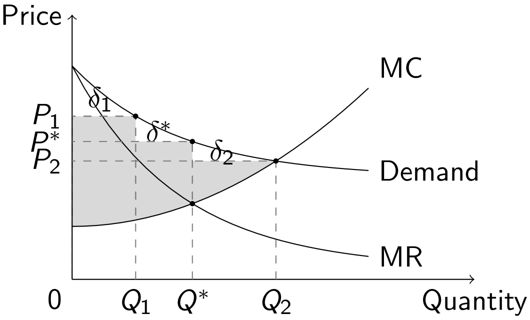


Illustration of Second Degree Price Discrimination

- Under the second-degree price discrimination $\{P_1, P^*, P_2\}$:
  - The monopolist's profit is illustrated by the shadow area, and the consumer surplus is $\delta_1 + \delta^* + \delta_2$.
- Under the monopoly pricing (without price discrimination):
  - The optimal monopoly price is P^* and the demand is Q^*;
  - The monopolist's profit is $P^* \cdot Q^*$, and the consumer surplus is $\delta_1 + \delta^* + (P_1 - P^*) \cdot Q_1$.



Second Degree Price Discrimination

- Comparison of Different Pricing Strategies
  - When the number of prices is one, the second-degree price discrimination degrades to the monopoly pricing;
  - When the price bundle curve approximates to the inverse demand curve $P(Q)$, the second-degree price discrimination converges to the first-degree price discrimination.

Third Degree Price Discrimination

- Limitation of First- and Second-Degree Price Discriminations
  - Needs the full or partial demand curve information of every individual consumer, and benefits from this information by charging the consumer different prices for different units of products.
- A Natural Question: *Whether (and how, if so) the monopolist discriminates the price, if he does not know the detailed demand curve information of each individual consumer, but knows from experience that different groups of consumers have different total demand curves?*

$\rightarrow$ **Third-Degree Price Discrimination**

Third Degree Price Discrimination

- With the **third-degree price discrimination** (or multi-market price discrimination), the monopolist specifies different prices for different consumer groups (with different total demand curves).
  - Example: The Disney Park offers different ticket prices to three player groups: children, adults, and elders.
- Third-degree price discrimination usually occurs when
  - the monopolist faces multiple identifiably different groups of consumers with different total demand curves;
  - the monopolist knows the total demand curve of every consumer group (but not the individual demand curve of each consumer.

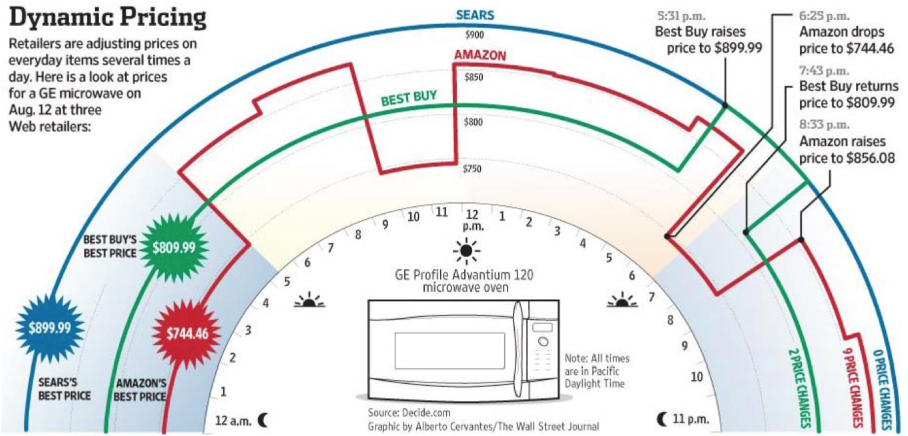
How to Identify Customers?

By Age

By Time

- Kindle 2
  - 02/2009: \$399
  - 07/2009: \$299
  - 10/2009: \$259
  - 06/2010: \$189

Even More Dynamic



More Innovative Ways

Third Degree Price Discrimination

- Consider a simple scenario:
  - Two groups (markets) of consumers:
  - The total demand curve in each market $i \in \{1, 2\}$ is $D_i(P)$;
  - The monopolist decides the price P_i for each market i.
- Key problem: *How should the monopolist set the prices $\{P_1, P_2\}$ to maximize his profit?*
  - Whether to charge the same price or different prices in different markets (groups)?
  - Which market should get the lower price if the monopolist charges different prices?
  - What is the relationship between the prices of two markets?

Third Degree Price Discrimination

- The monopolist's profit $\pi(P_1, P_2)$ under prices $\{P_1, P_2\}$ is

$$\pi(P_1, P_2) \triangleq P_1 \cdot Q_1 + P_2 \cdot Q_2 - C(Q_1 + Q_2)$$

- The first-order condition:

$$\frac{\partial \pi(P_1, P_2)}{\partial P_i} = Q_i + P_i \cdot \frac{\mathrm{d}Q_i}{\mathrm{d}P_i} - C'(Q_1 + Q_2) \cdot \frac{\mathrm{d}Q_i}{\mathrm{d}P_i} = 0$$

  - $Q_i \triangleq D_i(P_i)$ is the demand curve in market i;
  - $\eta_i \triangleq \frac{P_i}{Q_i} \frac{\mathrm{d}Q_i}{\mathrm{d}P_i}$ is the price elasticity of demand in market i;
  - $C'(Q_1 + Q_2)$ is the marginal cost (MC) of the monopolist;

Third Degree Price Discrimination

- The optimality condition:

$$C'(Q_1 + Q_2) = P_i + Q_i \cdot \frac{\mathrm{d}P_i}{\mathrm{d}Q_i} = P_i \cdot \left(1 - \frac{1}{|\eta_i|}\right)$$

$\Rightarrow$ Under the optimal prices (P_1^*, P_2^*), the marginal revenues (MR) in all markets are identical, and are equal to the marginal cost (MC):

$$P_1^* \cdot \left(1 - \frac{1}{|\eta_1|}\right) = P_2^* \cdot \left(1 - \frac{1}{|\eta_2|}\right)$$

  - $P_i \cdot (1 - \frac{1}{|\eta_i|})$ is the marginal revenue (MR) of the monopolist in market i;

Third Degree Price Discrimination

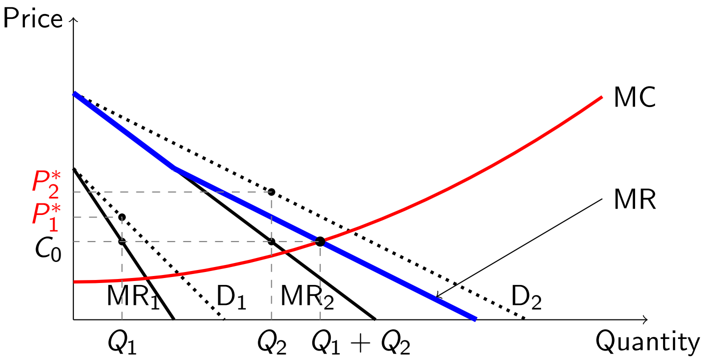
- The optimal prices (P_1^*, P_2^*) satisfy

$$P_1^* \cdot \left(1 - \frac{1}{|\eta_1|}\right) = P_2^* \cdot \left(1 - \frac{1}{|\eta_2|}\right)$$

  - If $|\eta_1| \neq |\eta_2|$, then $P_1^* \neq P_2^*$. That is, the monopolist will charge different prices when two markets have different price elasticities.
  - If $|\eta_1| > |\eta_2|$, then $P_1^* < P_2^*$. That is, the market with the higher price elasticity will get a lower optimal price.

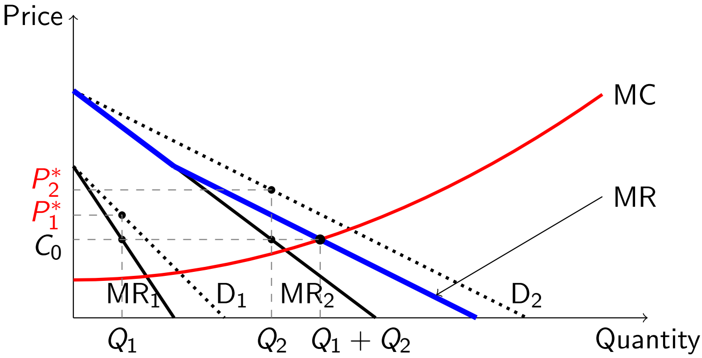
Third Degree Price Discrimination

- Graphic Interpretation of Optimal Prices (P_1^*, P_2^*)
  - Di: the demand curve in market i;
  - MRi: the marginal revenue curve in market i;
  - MR (the blue curve): the overall marginal revenue curve (summing MR1 and MR2 horizontally);
  - MC (the red curve): the marginal cost curve;



Third Degree Price Discrimination

- Graphic Interpretation of Optimal Prices (P_1^*, P_2^*)
  - Market 1: the demand is Q_1, the marginal revenue equals C_0;
  - Market 2: the demand is Q_2, the marginal revenue equals C_0;
  - Total market demand is $Q_1 + Q_2$, and the marginal cost is C_0;
  - C_0 is at the intersection of MC and MR curves.



Third Degree Price Discrimination

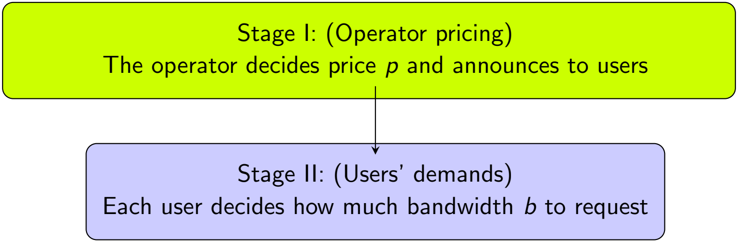
- Necessary conditions to make the third-degree price discrimination applicable and profitable:
  - Monopoly power: The firm must have the monopoly power to affect market price (there is no price discrimination in perfectly competitive markets).
  - Market segmentation: The firm must be able to split the market into different groups of consumers, and also be able to identify the type of each consumer.
  - Elasticity of demand: The price elasticities of demand in different markets are different.

Section 5.3: Cellular Network Pricing

Network Model

- A cellular operator with B Hz of bandwidth
- Sell bandwidth to multiple users

Two-Stage Decision Process



User's Spectrum Efficiency

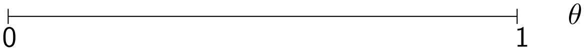
- h: a user's (average) channel gain between him and the base station
- P: a user's transmission power density (per unit bandwidth)
- θ: a user's spectrum efficiency (data rate per unit bandwidth)

$$\theta = \log_2(1 + \mathtt{SNR}) = \log_2\left(1 + \frac{Ph}{n_0}\right)$$

- When allocated bandwidth b, the user achieves a data rate of θb

User's Spectrum Efficiency

- Different users have different spectrum efficiencies
  - Due to different values of P and h
  - Indoor users often have a smaller h than outdoor users
- Normalize the range of θ to be [0,1]
  - Divided by the maximum value of θ among all users



User's Utility and Payoff

- A user's utility when allocated bandwidth b

$$u(\theta, b) = \ln(1 + \theta b)$$

- A user's payoff under linear pricing p:

$$\pi(\theta, b, p) = \ln(1 + \theta b) - pb$$

User's Demand in Stage II

- Payoff maximization problem

$$\max_{b\geq 0} \pi(\theta, b, p) = \max_{b\geq 0} \left(\ln(1+\theta b) - pb\right)$$

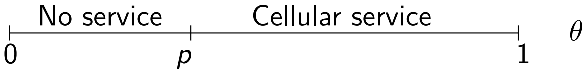
- Concave maximization problem $\Rightarrow$ user's optimal demand

$$b^*(\theta, p) = \begin{cases} \frac{1}{p} - \frac{1}{\theta}, & \text{if } p \leq \theta, \\ 0, & \text{otherwise.} \end{cases}$$

- User's maximum payoff

$$\pi(\theta, b^*(\theta, p), p) = \begin{cases} \ln\left(\frac{\theta}{p}\right) - 1 + \frac{p}{\theta}, & \text{if } p \leq \theta, \\ 0, & \text{otherwise.} \end{cases}$$

User Separation Based on Spectrum Efficiency



Users' Total Demand

- Price $p \leq \max_{\theta \in [0,1]} \theta = 1$
  - If $p > 1$, the total user demand will be 0
- Total user demand

$$Q(p) = \int_p^1 \left(\frac{1}{p} - \frac{1}{\theta} \right) d\theta = \frac{1}{p} - 1 + \ln p$$

  - Decreasing in p.

Operator's Optimal Pricing

- Operator's revenue maximization problem

$$\max_{0<p\leq 1} \min\left(pB, pQ(p)\right)$$

  - pB is increasing in p
  - $pQ(p)$ is decreasing in p:

$$\frac{\mathrm{d}pQ(p)}{\mathrm{d}p} = \ln p < 0$$

  - We can show that at the optimal price p^*, $p^*B = p^*Q(p^*)$.

Operator's Optimal Pricing

- Operator's revenue maximization problem

$$\max_{0<p\leq 1} \min\left(pB, pQ(p)\right)$$

  - pB is increasing in p
  - $pQ(p)$ is decreasing in p:

$$\frac{\mathrm{d}pQ(p)}{\mathrm{d}p} = \ln p < 0$$

  - We can show that at the optimal price p^*, $p^*B = p^*Q(p^*)$.

- The optimal price p^* is the unique solution of

$$B = \frac{1}{p^*} - 1 + \ln p^*$$

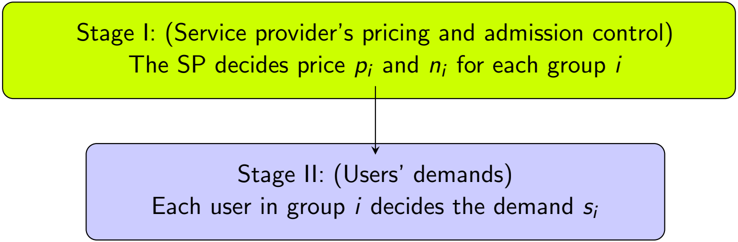
  - $B \to 0 \Rightarrow p \to 1$
  - $B \to \infty \Rightarrow p \to 0$

Section 5.4: Partial Price Differentiation

Network Model

- One wireless service provider (SP)
- A set of $\mathcal{I}$ groups of users, where each group $i \in \mathcal{I}$ has
  - N_i homogenous users
  - Same utility function $u_i(s_i) = \theta_i \ln(1 + s_i)$
  - Groups have decreasing preference coefficients: $\theta_1 > \theta_2 > \cdots > \theta_I$
- The SP's decision for each group i
  - Admit $n_i \leq N_i$ users
  - Charge a unit price p_i (per unit of resource)
  - Subject to total resource limit: $\sum_i n_i s_i \leq S$

Two-Stage Decision Process



- Complete price differentiation: charge up to I different prices
- Single pricing (no price differentiation): charge one price
- Partial price differentiation: charge J prices with $1 \leq J \leq I$

Complete Price Differentiation: Stage II

- Each (admitted) group i user chooses s_i to maximize payoff

$$\underset{s_i \geq 0}{\text{maximize}} \; (\theta_i \ln(1 + s_i) - p_i s_i)$$

- The unique optimal demand is

$$s_i^*(p_i) = \max\left(\frac{\theta_i}{p_i} - 1, 0\right) = \left(\frac{\theta_i}{p_i} - 1\right)^+$$

Complete Price Differentiation: Stage I

- SP performs admission control $\boldsymbol{n}$ and determines prices $\boldsymbol{p}$:

$$
\begin{aligned}
\underset{\boldsymbol{n},\boldsymbol{p}\geq 0,\boldsymbol{s}\geq 0}{\text{maximize}} \quad & \sum_{i\in\mathcal{I}} n_i p_i s_i \\
\text{subject to} \quad & s_i = \left(\frac{\theta_i}{p_i} - 1\right)^+, \;\; i \in \mathcal{I}, \\
& n_i \in \{0,\ldots,N_i\} \;,\;\; i \in \mathcal{I}, \\
& \sum_{i\in\mathcal{I}} n_i s_i \leq S.
\end{aligned}
$$

  - The Stage II's user responses are incorporated

Complete Price Differentiation: Stage I

- SP performs admission control $\boldsymbol{n}$ and determines prices $\boldsymbol{p}$:

$$\begin{aligned} \underset{\boldsymbol{n},\boldsymbol{p}\geq 0,\boldsymbol{s}\geq 0}{\text{maximize}} \quad & \sum_{i\in\mathcal{I}} n_i p_i s_i \\ \text{subject to} \quad & s_i = \left(\frac{\theta_i}{p_i} - 1\right)^+, \ \ i \in \mathcal{I}, \\ & n_i \in \{0, \ldots, N_i\} \ , \ \ i \in \mathcal{I}, \\ & \sum_{i\in\mathcal{I}} n_i s_i \leq S. \end{aligned}$$

  - The Stage II's user responses are incorporated

- This problem is challenging to solve due to non-convex objectives, integer variables, and coupled constraint.

Complete Price Differentiation: Stage I

- The admission control and pricing can be decoupled

- At the unique optimal solution
  - Do not reject any user
  - Charge prices such that users perform voluntary admission control: there exists a group threshold K^{cp} and λ^{cp} with

$$p_i^* = \begin{cases} \sqrt{\theta_i \lambda^*}, & i \leq K^{cp}; \\ \theta_i, & i > K^{cp}. \end{cases}$$

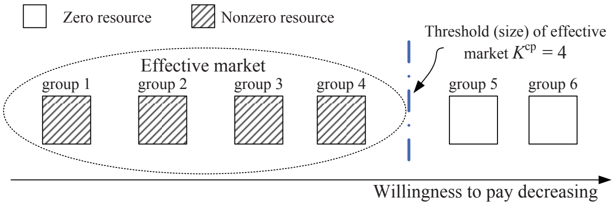
and

$$s_i^* = \begin{cases} \sqrt{\frac{\theta_i}{\lambda^*}} - 1, & i \leq K^{cp}; \\ 0, & i > K^{cp}. \end{cases}$$

  - The choice of λ^* satisfies

$$\sum_{i=1}^{K^{cp}} n_i \left(\sqrt{\frac{\theta_i}{\lambda^*}} - 1 \right) = S$$

Complete Price Differentiation: Optimal Solution

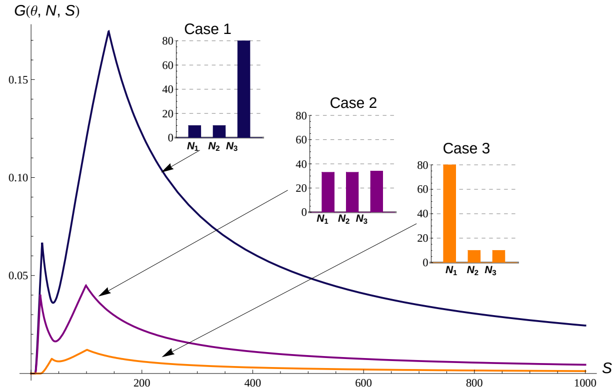


- Effective market: includes groups receiving positive resources

Single Pricing (No Price Differentiation)

- Problem formulation similar as the complete price differentiation case
- Key difference: change the same price p to all groups
- Similar optimal solution structure
- Effective market is no larger than the one under complete price differentiation
  - Less users will be served

Effectiveness of Complete Price Differentiation



- Revenue gain of price differentiation is the largest when
  - The high willingness-to-pay users are minority, and
  - Total resource S is limited

Partial Price Differentiation

- The most general case
- SP can charge J prices to I groups, where $J \leq I$
  - Complete price differentiation: $J = I$
  - Single pricing: $J = 1$
- How to divide I groups into J clusters, and optimize the J prices?

Partial Price Differentiation

- $\boldsymbol{a}=\{a_i^j,\ j \in \mathcal{J},\ i \in \mathcal{I}\}$: binary variables defining the *partition*
  - $a_i^j = 1 \Rightarrow$ group i is in cluster j
- Revenue optimization problem:

$$
\begin{aligned}
\underset{\{n_i, p_i, s_i, p^j, a_i^j\}_{\forall i,j}}{\text{maximize}} \quad & \sum_{i \in \mathcal{I}} n_i p_i s_i \\
\text{subject to} \quad & s_i = \left(\frac{\theta_i}{p_i} - 1\right)^+, \ \forall\, i \in \mathcal{I}, \\
& n_i \in \{0, \ldots, N_i\},\ \forall\, i \in \mathcal{I}, \\
& \sum_{i \in \mathcal{I}} n_i s_i \le S, \\
& p_i = \sum_{j \in \mathcal{J}} a_i^j p^j, \\
& \sum_{j \in \mathcal{J}} a_i^j = 1,\ a_i^j \in \{0, 1\}, \forall\, i \in \mathcal{I}.
\end{aligned}
$$

Three-Level Decomposition

- Level-1 (Cluster Partition): partition I groups into J clusters
- Level-2 (Inter-Cluster Resource Allocation): allocate resources among clusters (subject to the total resource constraint)
- Level-3 (Intra-Cluster Pricing and Resource Allocation): optimize pricing and resource allocations within each cluster

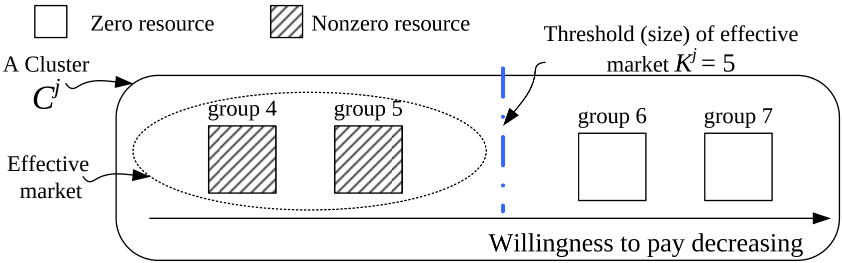
Level 3: Pricing and Resource Allocation in Single Cluster

- Given a fixed partition $\boldsymbol{a}$ and a cluster resource allocation $\boldsymbol{s} \triangleq \{s^j\}_{j\in\mathcal{J}}$
- Solve the pricing and resource allocation problems in cluster $\mathcal{C}^j$:

$$\begin{aligned} \text{Level-3:} \quad & \underset{n_i, s_i, p^j}{\text{maximize}} && \sum_{i\in\mathcal{C}^j} n_i p^j s_i \\ & \text{subject to} && s_i = \left(\frac{\theta_i}{p^j} - 1\right)^+, \quad \forall\, i \in \mathcal{C}^j, \\ &&& n_i \le N_i, \quad \forall\, i \in \mathcal{C}^j, \\ &&& \sum_{i\in\mathcal{C}^j} n_i s_i \le s^j. \end{aligned}$$

- Equivalent to a single pricing problem

Level 3: Effective Market in a Single Cluster



Level 2: Resource Allocation Among Clusters

- For a fixed partition $\boldsymbol{a}$
- Consider the resource allocation among clusters:

$$\begin{aligned} \text{Level-2:} \quad & \underset{s^j \geq 0}{\text{maximize}} && \sum_{j \in \mathcal{J}} R^j(s^j, \boldsymbol{a}) \\ & \text{subject to} && \sum_{j \in \mathcal{J}} s^j \leq S. \end{aligned}$$

- Solving Level 2 and Level 3 together is equivalent of solving a complete price differentiation problem

Level-1: Cluster Partition

$$\text{Level-1:} \quad \underset{a_i^j \in \{0,1\}, \forall i,j}{\text{maximize}} \quad R_{pp}(\boldsymbol{a})$$

$$\text{subject to} \quad \sum_{j \in \mathcal{J}} a_i^j = 1, \; i \in \mathcal{I}.$$

How to Perform Cluster Partition in Level 1

- Naive exhaustive search leads to formidable complexity for Level 1

Groups	$I = 10$		$I = 100$	$I = 1000$
Clusters	$J = 2$	$J = 3$	$J = 2$	$J = 2$
Combinations	511	9330	6.33825×10^{29}	5.35754×10^{300}

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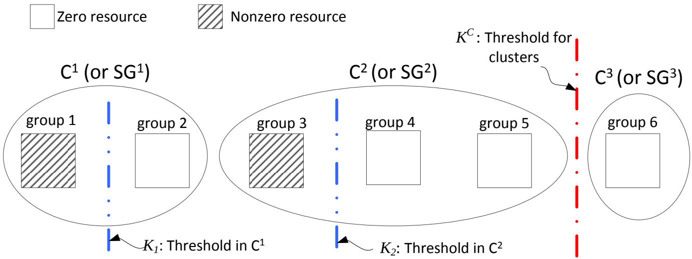
- Do we need to check all partitions?

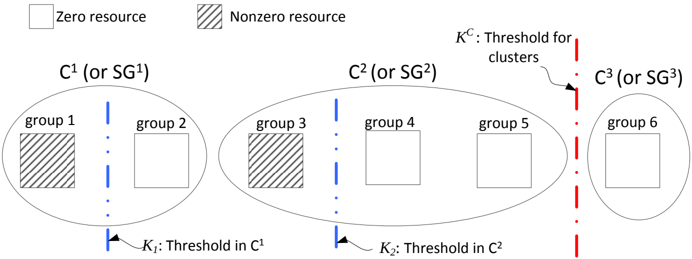
Property of An Optimal Partition

- Will the following partition ever be optimal?



Property of An Optimal Partition

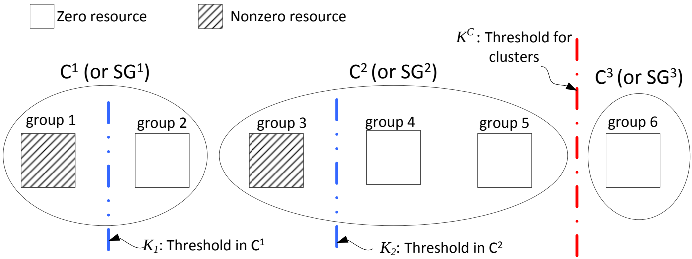
- Will the following partition ever be optimal?



- No.

Property of An Optimal Partition

- Will the following partition ever be optimal?



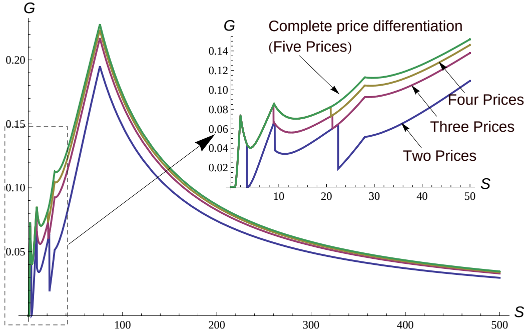
- No.
- We prove that group indices in the effective market are consecutive.

Reduced Complexity of Cluster Partition in Level I

- The search complexity reduces to polynomial in I.

Groups	$I = 10$		$I = 100$	$I = 1000$
Clusters	$J = 2$	$J = 3$	$J = 2$	$J = 2$
Combinations	511	9330	6.33825×10^{29}	5.35754×10^{300}
Reduced Combos	9	36	99	999

Relative Revenue Gain



- A total of $I = 5$ groups
- Plot the relative revenue gain of price differentiation vs. total resource
- Maximum gains in the small plot
  - $J = 3$ is the sweet spot

Section 5.5: Chapter Summary

Key Concepts

- Theory
  - Monopoly pricing and the demand elasticity
  - First-degree price discrimination
  - Second-degree price discrimination
  - Third-degree price discrimination
- Application
  - Cellular Network Pricing
  - Partial Price Discrimiation

References and Extended Reading

L. Duan, J. Huang, and B. Shou, "Economics of Femtocell Service Provisions," *IEEE Transactions on Mobile Computing*, vol. 12, no. 11, pp. 2261 - 2273, November 2013

S. Li and J. Huang, "Price Differentiation for Communication Networks," *IEEE Transactions on Networking*, vol. 22, no. 2, pp. 703 - 716, June 2014

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