Wireless Network Pricing
Chapter 7: Network Externalities

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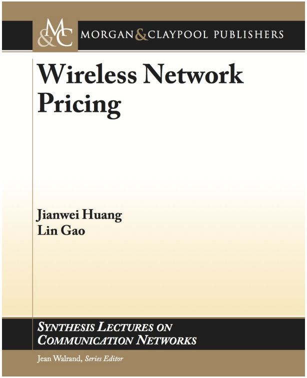
The Book



- E-Book freely downloadable from NCEL website: http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing
- Physical book available for purchase from Morgan & Claypool (http://goo.gl/JFGlai) and Amazon (http://goo.gl/JQKaEq)

Chapter 7: Network Externalities

Section 7.1: Theory: Network Externalities

What is Externality?

Definition (Externality)

An externality is any side effect (benefit or cost) that is imposed by the actions of a player on a third-party not directly involved.

Examples: Negative Externality

Air Pollution (source: Internet)

Examples: Negative Externality

Second-hand Smoke (source: Internet)

Examples: Negative Externality

Traffic Congestion (source: Internet)

Examples: Positive Externality

Lighthouse (source: Internet)

Examples: Positive Externality

Bee Keeping (source: Internet)

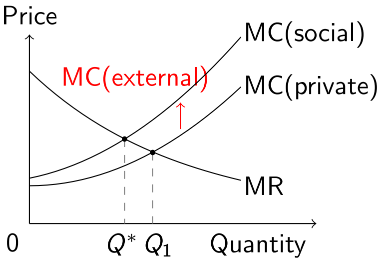
Examples: Positive Externality

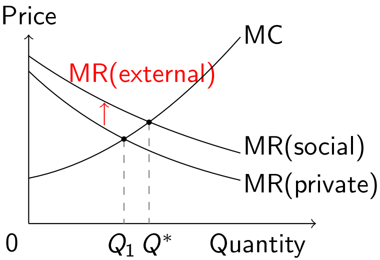
Immunization (source: Internet)

Impact of Externality

- Can cause market failure without proper prices
  - The market outcome will no longer be efficient.
  - If market prices do not reflect the costs or benefits of externalities.
- Example: negative externality of pollution
  - The market price for steel reflects the cost labor, capital, and other inputs, but may not include the cost due to air pollution.
  - The steel manufacturer may produce more products than the socially optimal level.

Graphical Illustration of Market Failure

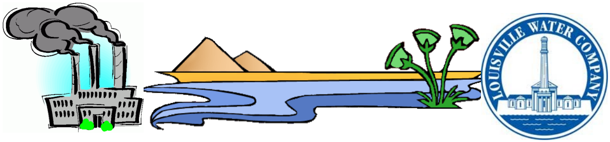




- Social optimal production level Q^*:
  - Social Marginal Cost (MC) = Social Marginal Revenue (MR)
- Left: negative production externality
  - Private MC < Social MC
  - Local optimal quality $Q_1 >$ Social optimal quality Q^*
- Right: positive consumption externality
  - Private MR < Social MR
  - Local optimal quality $Q_1 <$ Social optimal quality Q^*

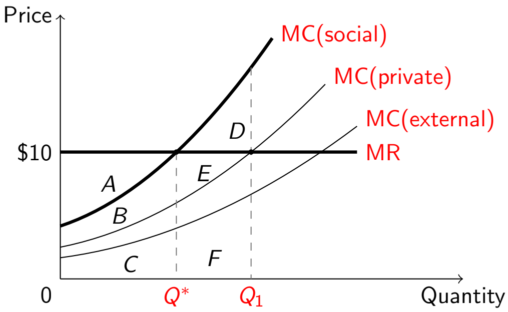
Negative Network Externality

A Case Study: Water Pollution



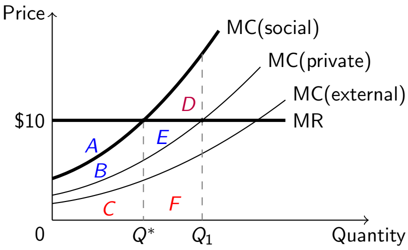
- The chemical company produces chemical products and discharges wastewater into the river.
- The water company produces bottle water by drawing water from the river.
- Water pollution increases the production cost of the water company.

Graphical Illustration



- Constant MR per chemical product: $10.
- Social MC = private MC (chemical plant) + external MC (pollution)
- Social optimal quant $Q^* <$ local optimal quality Q_1

At Local Optimal Quality Q_1



- The chemical plant's profit (i.e., revenue - cost):

$$\int_0^{Q_1} (MR - MCPrivate(Q))\ \mathrm{d}Q = A + B + E$$

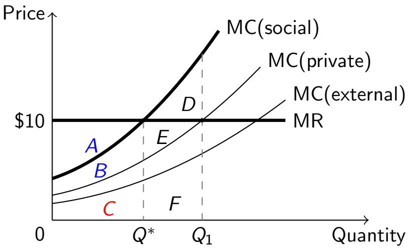
- The water company's profit due to externality (assuming 0 revenue):

$$-\int_0^{Q_1} (MCExternal(Q))\ \mathrm{d}Q = -(C + F)$$

- Since $C = B$ and $F = D + E$, the social surplus (sum of two profits):

$$A + B + E - (C + F) = A - D$$

At Social Optimal Quality Q^*



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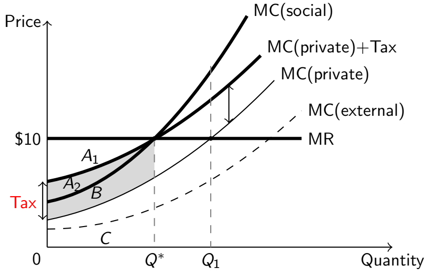
- Since $C = B$, the social surplus (sum of two profits):

$$A + B - C = A$$

Comparison

- Social suplus at $Q_1 : A - D$
- Social surplus at $Q^* : A$
- With negative externally, individual profit maximization hurts the social surplus
- Solution: Pigovian tax

Pigovian Tax



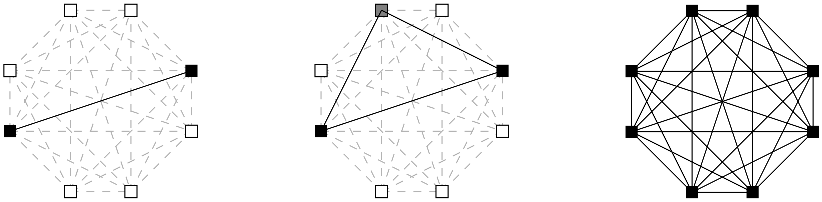
- Charge chemical plant a tax
  - Tax = external marginal cost at the optimal solution Q^*
- Individual profit maximisation leads to production level of Q^*
  - Chemical plant profit $= \int_0^{Q^*} (MR - MCPrivate(Q) - Tax)\, \mathrm{d}Q = A_1$

The Coase Theorem

- Nobel Laureate Ronald Coase proposes another view of externality
- Assumptions: Transaction cost is negligible, property rights are clear
- Result: Trade in externality will lead to efficient use of the resource
- Back to the previous example
  - If water company owns the water: it can charge the chemical plant a price equal to the negative externally
  - If chemical plant owns the water: it can demand a compensation from water company for reducing the chemical production quantity
  - Either way, it is possible to maximize social surplus

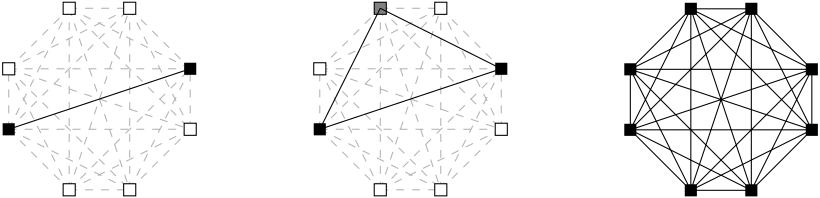
Positive Network Externality

A Case Study: Network Effect



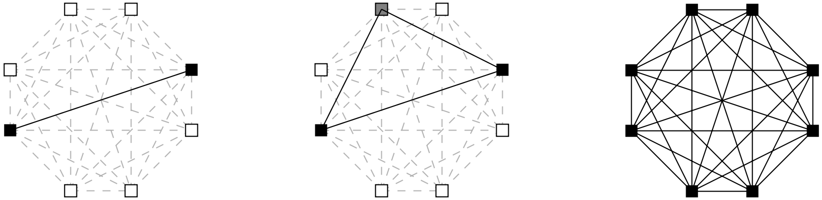
- More usage of the product by any user increases the product's value for other users.

Metcalfe's Law



- Consider a network of N users.
- Each user perceives a value increasing in N.
- Each user attaches the same value to the possibility of connecting with any one of the other $N-1$ users.
- Total network value $N(N-1) \approx N^2$.

Briscore's Refinement



- Each user ranks other users in terms of decreasing importance.
- Attach a value of $1/k$ to the k_{th} important neighbour.
- Total network value $N\left(\sum_{k=1}^{N-1} 1/k\right) \approx N \log N$.

Different Types of Network Effect

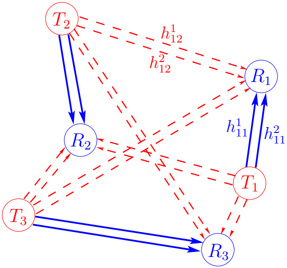
- Direct network effect: telephone, online social network
- Indirect network effect: Office for Windows, DVDs for DVD players
- Local network effect: instant messaging

Section 7.2: Distributed Wireless Interference Compensation

Wireless Power Control

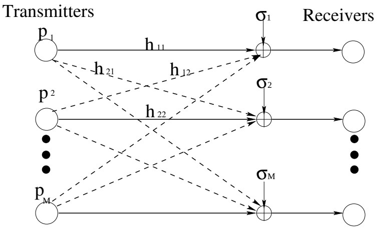
- Distributed power control in wireless ad hoc networks
- Elastic applications with no SINR targets
- Want to maximize the total network performance

Network Model



- Single-hop transmissions.
- A user = a transmitter/receiver pair.
- Transmit over multiple parallel channels.
- Interferences in the same channel (negative externality).
- We focus on single channel here.

Single Channel Communications



- A set of $\mathcal{N} = \{1, ..., n\}$ users.
- For each user $n \in \mathcal{N}$:
  - Power constraint: $p_n \in [P_n^{min}, P_n^{max}]$.
  - Received SINR (signal-to-interference plus noise ratio):

$$\gamma_n = \frac{p_n h_{n,n}}{\sigma_n + \sum_{m \neq n} p_m h_{n,m}}.$$

  - Utility function $U_n(\gamma_n)$: increasing, differentiable, strictly concave.

Network Utility Maximization (NUM) Problem

NUM Problem

$$\max_{\{P_n^{\min} \leq p_n \leq P_n^{\max}, \forall n\}} \sum_n U_n(\gamma_n).$$

- Technical Challenges:
  - Coupled across users due to interferences.
  - Could be non-convex in power.
- We want: efficient and distributed algorithm, with limited information exchange and fast convergence.

Benchmark - No Information Exchange

- Each user picks power to maximize its own utility, given current interference and channel gain.
- Results in $p_n = P_n^{max}$ for all n.
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- We propose algorithm with limited information exchange.
  - Have nice interpretation as distributed Pigovian taxation.
  - Analyze its behavior using supermodular game theory.

ADP Algorithm: Asynchronous Distributed Pricing

- Price Announcing: user n announces "price" (per unit interference):

$$\pi_n = \left| \frac{\partial U_n(\gamma_n)}{\partial I_n} \right| = \frac{\partial U_n(\gamma_n)}{\partial \gamma_n} \frac{\gamma_n^2}{p_n h_{n,n}}.$$

- Power Updating: user n updates power p_n to maximize surplus:

$$S_n = U_n(\gamma_n) - p_n \sum_{m \neq n} \pi_m h_{m,n}.$$

- Repeat two phases asynchronously across users.
- Scalable and distributed: only need to announce single price, and know limited channel gains ($h_{m,n}$).

ADP Algorithm

- Interpretation of prices: Pigovian taxation

ADP Algorithm

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- ADP algorithm: distributed discovery of Pigovian taxes
  - When does it converge?
  - What does it converge to?
  - Will it solve **NUM** Problem ?
  - How fast does it converge?

Convergence

- Depends on the utility functions.

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- Coefficient of relative Risk Aversion (CRA) of $U(\gamma)$:

$$CRA(\gamma) = -\frac{\gamma U''(\gamma)}{U'(\gamma)}.$$

  - larger CRA $\Rightarrow$ "more concave" U.

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- **Theorem**: If each user n has a positive minimum transmission power and $CRA(\gamma_n) \in [1, 2]$, then there is a unique optimal solution of **NUM** Problem, and the ADP algorithm globally converges to it.
- Proof: relating this algorithm to a fictitious supermodular game.

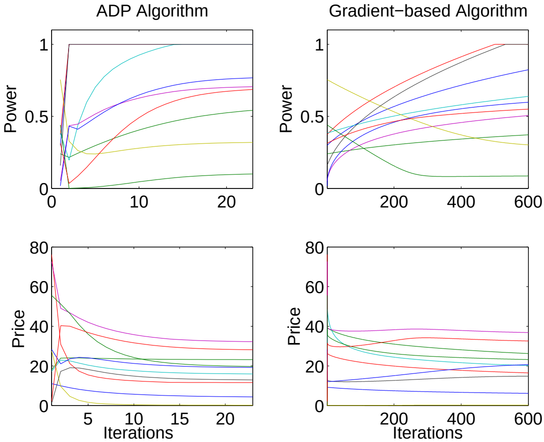
Supermodular Games

- A class of games with strategic complementaries
  - Strategy sets are compact subsets of $\mathbb{R}$; and each player's pay-off S_n has increasing differences:

$$\frac{\partial^2 S_n}{\partial x_n \partial x_m} > 0, \forall n, m.$$

- Key properties:
  - A PNE exists.
  - If the PNE is unique, then the asynchronous best response updates will globally converge to it.

Convergence Speed



- 10 users, log utilities.
- ADP algorithm (left figures) converges much faster than a gradient-based method (right figures).

Section 7.3: 4G Network Upgrade

When To Upgrade From 3G to 4G?

- Early upgrade:
  - More expensive, as cost decreases over time
  - Starts with few users, hence a small initial revenue
- Late upgrade:
  - Leads to a smaller market share
  - Delays 4G revenues
- Need to
  - Capture the above tradeoffs
  - Consider the dynamics of users adopting 4G and switching providers
  - Understand the upgrade timing between competing cellular providers

Duopoly Model

- Two competing operators
  - Initially both using 3G technology
  - Operator i decides to upgrade to 4G at time T_i
  - Each operator wants to maximize its long-term profit
- What will be the equilibrium of (T_1^*, T_2^*)?

Users Switching

- W.L.O.G., assume $T_1 < T_2$
- Three time periods: $[0, T_1]$, $(T_1, T_2]$, and (T_2, ∞)

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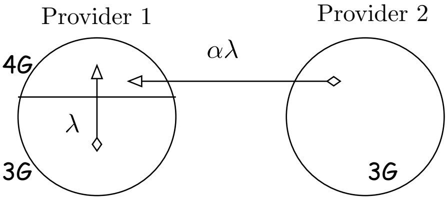
Users Switching

- When $t \in (T_1, T_2]$: both inter- and intra- operator user switching



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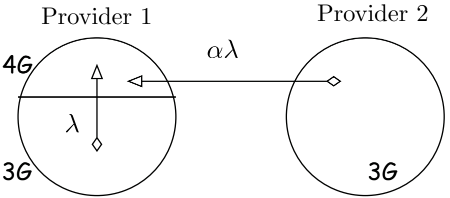
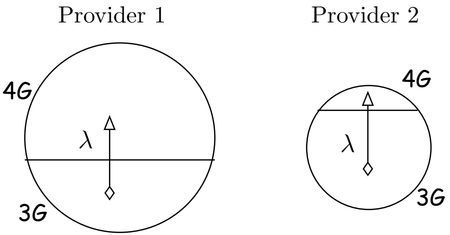


- When $t \in (T_2, \infty)$: only intra-operator user switching



Network Value (Revenue)

- Network value depends on the number of subscribers
  - Assume that operator i has N_i 4G users, $i = 1, 2$
  - Total 4G network value is $(N_1 + N_2)\log(N_1 + N_2)$ (network effect)
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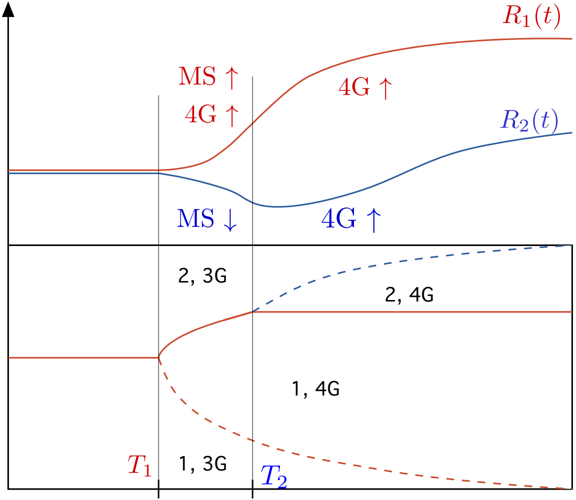
- Later upgrade $\Rightarrow$ take advantage of existing 4G population
- The revenue for 3G network is similar, with an coefficient $\gamma \in (0, 1)$

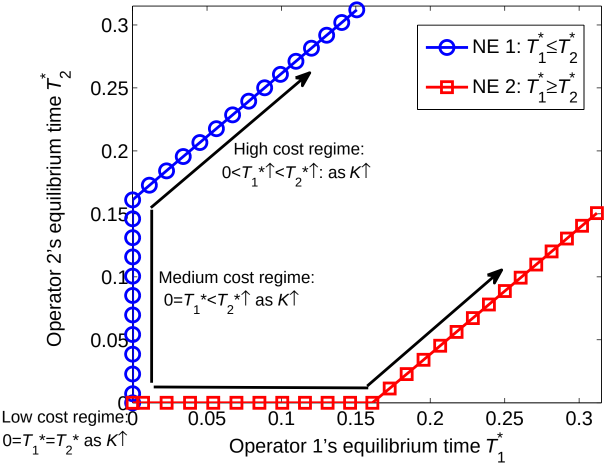
Revenue and Market Share



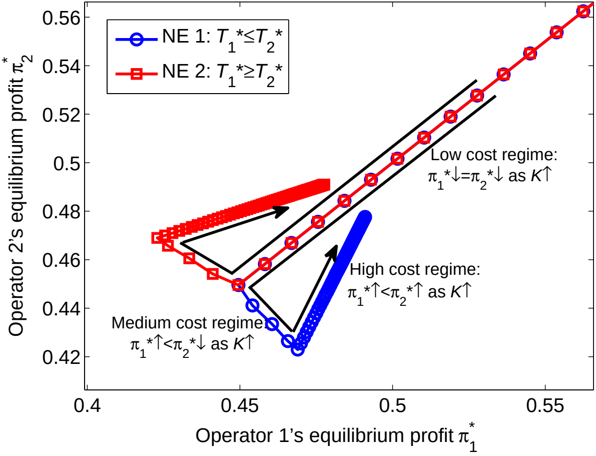
Upgrade Cost and Time Discount

- One-time upgrade cost:
  - K at time $t = 0$
  - Discounted over time: $K \exp(-Ut)$
- Revenue is also discounted over time by $\exp(-St)$
- Earlier upgrade $\Rightarrow$ larger revenue and larger cost

Equilibrium Timings



Equilibrium Profits



Section 7.4: Chapter Summary

Key Concepts

- Theory
  - Positive and negative Externality
  - Market failure
  - Pigovian tax
  - Network effect
- Application
  - Distributed wireless power control based on Pigovian tax
  - Cellular network upgrade considering network effect

References and Extended Reading

- J. Huang, R. Berry and M. Honig, "Distributed Interference Compensation for Wireless Networks," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 5, pp. 1074-1084, 2006
- L. Duan, J. Huang, and J. Walrand, "Economic Analysis of 4G Network Upgrade," *IEEE Transactions on Mobile Computing*, accepted 2014

http://ncel.ie.cuhk.edu.hk/content/wireless-network-pricing