Dynamic Auction: Mechanisms and Applications

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Outline

- Preliminary
- Benchmark Model
- Extensions
- Applications
- Conclusion

What is an Auction ?

- An auction is a market institution with an explicit set of rules (mechanisms) determining resource allocation and prices on the basis of bids from the market participants.
- Auctioneer and Bidder

Who is in an Auction ?

- 1 seller and N buyers (Monopoly-Oligopsony)
- 1 buyer and M sellers (Monopsony-Oligopoly) (reverse auction)
- M sellers and N buyers (Oligopoly-Oligopsony) (competitive setting)

Seller\Buyer	Monopsony	Oligopsony	Perfect Competition
Monopoly	$\{1, 1\}$	$\{1, N\}$	$\{1, \infty\}$
Oligopoly	$\{M, 1\}$	$\{M, N\}$	$\{M, \infty\}$
Perfect Competition	$\{\infty, 1\}$	$\{\infty, N\}$	$\{\infty, \infty\}$

What kinds of goods are sold ?

- Indivisible goods (e.g., antiques, artworks, cars)
- Divisible goods with fixed fragment size (e.g., a box of wines)
- Divisible goods with arbitrary fragment size (e.g., transmission power)

Why is an Auction used ?

- Some products (goods or items for sale) have no standard value.
- Information Asymmetry - The seller does not know any bidder's valuation of the item for sale, or the buyer does not know any seller's producing cost of the item (in reverse auction).

Auction Mechanism

- Bidding Process - how to bid, e.g., how much to bid, whether to repeatedly bid (open-bid) or just bid once (sealed-bid), etc.
- Assigning Process - how to assign the items among bidders—that is, who are the winning bidders?
- Pricing Process - how to charge the winning bidders —that is, how much should the winners pay?

Private vs. Common Values

- Independent-private values - Any bidder's valuation for an item is independent from others. Every bidder knows his true valuation while does not know anyone else's.
- Common values - The item has a common objective value for all bidders. Every bidder does not know the true value, but has a guess on it.
- Correlated values - Any bidder's valuation for an item is comprised by a common factor and a independent private factor.

Open vs. Sealed-bid

- Open Format - Bidders bid openly and repeatedly, and each subsequent bid is related to the previous bid.
(Also named as open/progressive/dynamic auction)

↕ Outcome Equivalence

- Sealed-bid Format - All bidders simultaneously submit sealed bids so that no bidder knows the bid of any other participant.
(Also named as close/static auction)

In this ppt, we focus on open format
——Dynamic Auction.

Benchmark Model

Benchmark Model

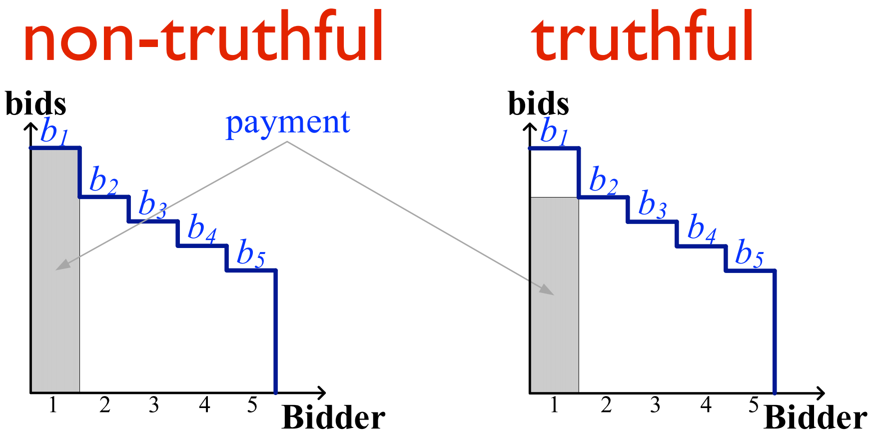
- 1 seller (auctioneer) with 1 unit of indivisible item for sale.
- N potential buyers (bidders) with independent-private values.
- All participants are risk neutral.

Four Basic Auction Types

- First-price Sealed-Bid Auction
- Second-price Sealed-Bid Auction
- The English Auction
- The Dutch Auction

Sealed-bid Auction - A review

- First-price Sealed-Bid Auction - (i) Bidders submit sealed bids simultaneously; (ii) winner is the highest bidder; (iii) winner pays a price equal to his bid.
- Second-price Sealed-Bid Auction - (i) Bidders submit sealed bids simultaneously; (ii) winner is the highest bidder; (iii) winner pays a price equal to the second highest bid.



(a) First-price (b) Second-price

Pricing Mechanism in Single-unit Sealed-bid auctions

The English Auction

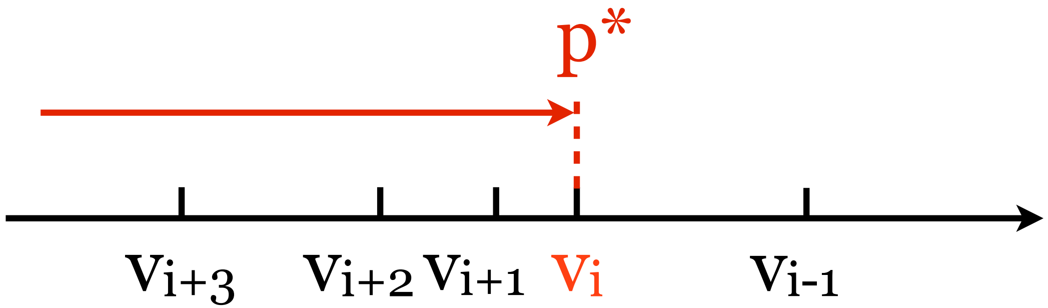
- The auctioneer calls an initial low price and then gradually raises the price until there remains only one bidder.
  (Also named as *open ascending price auction*)
- The second-last bidder will drop out of the bidding as soon as the price exceeds his own valuation. The highest valuation bidder wins and pays a price equal to the second highest valuation.

Yields the same outcome as Second-price sealed-bid auction !

The English Auction

- p : the price a bidder i will drop out
- Best Strategy for bidder i :

$$p^* = v_i$$



The Dutch Auction

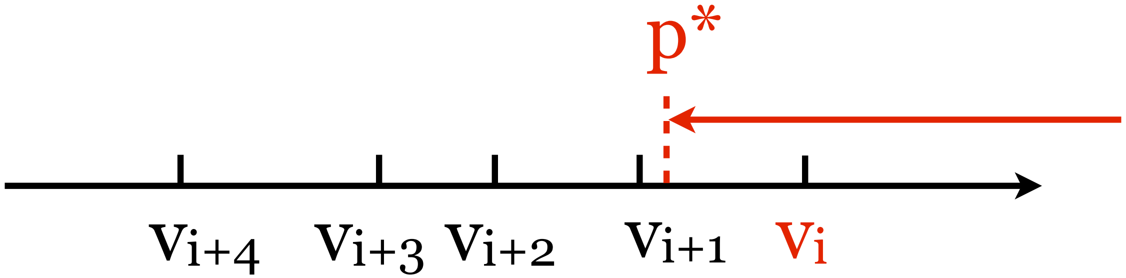
- The auctioneer calls an initial high price and then gradually lowers the price until one bidder accepts the current price.
  (Also named as *open descending price auction*)
- Each bidder must strategically decide at which price he is going to accept.
  Tradeoff : accepting at a high price implies high win-probability but low utility.

Strategically equivalent to First-price sealed-bid auction !!

The Dutch Auction

- p : the price a bidder i is going to accept
- Best Strategy for bidder i :

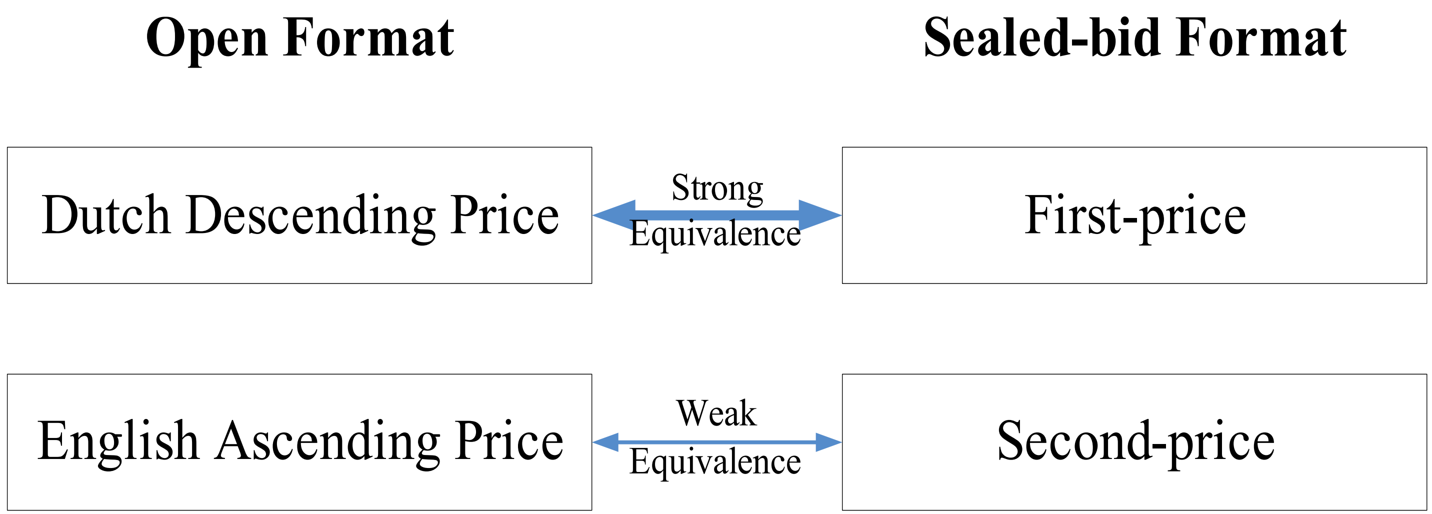
$$p^* = \max_p (v_i - p)Pr(p)$$



- Example: N bidders, i.i.d. valuations,

$$p^* = v_i * (N-1)/N$$

Outcome Equivalence



Revenue Equivalence Theorem

- For the benchmark model, each of the English auction, the Dutch auction, the first-price sealed-bid auction and the second-price sealed-bid auction yields the same price on average. (Vickrey 1961)

- Revenue equivalence does not hold if participants are not risk neutral !!

Extensions

Extension 1 - Double Market

- M sellers each with 1 unit of indivisible item.
- Reserve-prices for sellers are independent and private.
- N potential buyers (bidders) with independent-private values.
- All items are identical.
- All participants are risk neutral.

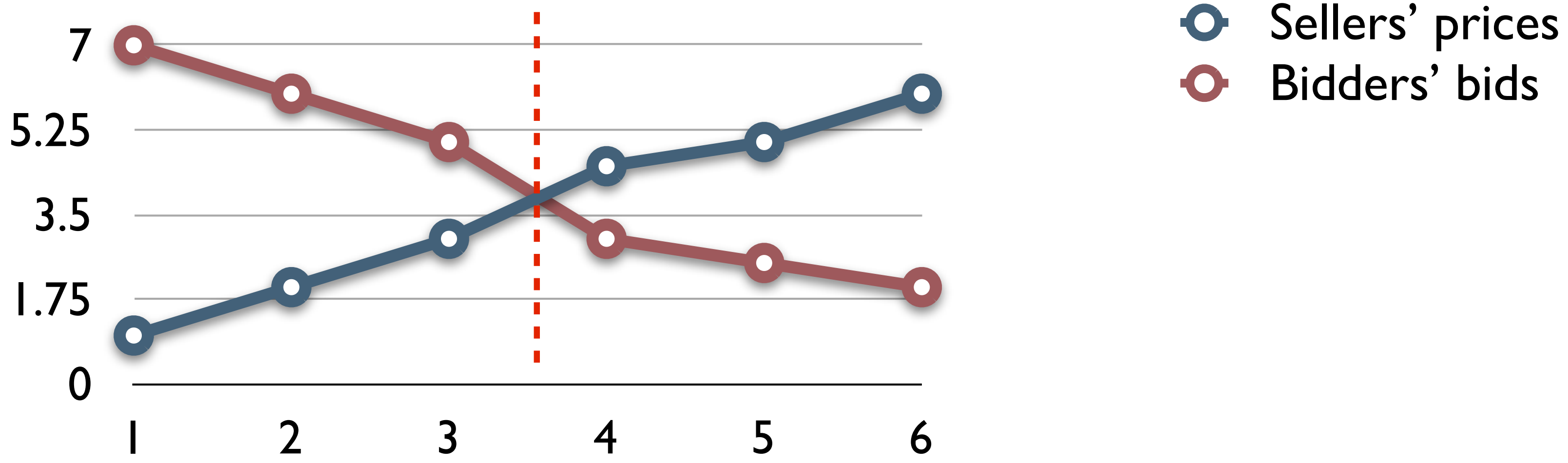
Two Double Auction Types

- Double Sealed-bid Auction
- Simplified Walrasian Auction

Double sealed-bid Auction - A review

- (i) Buyers submit sealed bids;
- (ii) Sellers submit sealed bids;
- (iii) Auctioneer chooses price p that clears the market;
- (iv) All the sellers who bid less than p sell his item at price p; All the buyers who bid more than p buy an item at price p.

truthful



Simplified Walrasian Auction

- The auctioneer calls an initial price and then gradually raises/decreases the price if demand is larger/smaller than supply, until the market is clear (demand is equal to supply).

Yields the same outcome as the Double auction !

Extension 2 - Multiple Units

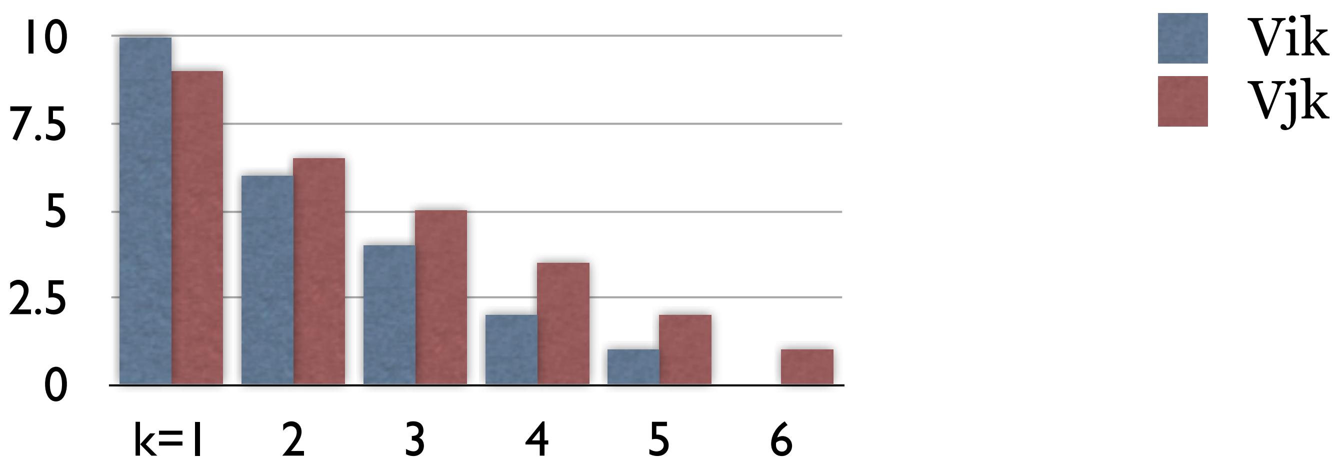
- 1 seller with K units of indivisible items for sale.
- N potential buyers (bidders) with independent-private values.
- Every buyer requires 1~K units of items.
- All items are identical.
- All participants are risk neutral.

Decreasing Marginal Utility

- Every buyer requires $1 \sim K$ units of items. Let v_{ik} denotes the marginal utility by the k-th item the buyer i gets. Valuation of any buyer i is represent by a vector:

$$\mathbf{v}_i = (v_{i1}\ v_{i2} \ldots v_{iK})$$

- The marginal utility by an extra item is decreasing with the existing number of items the buyer gets (concave utility).



Basic Multi-Unit Auction Types

- Discriminatory Auction
- Uniform-price Auction
- Vickery Auction
- Multi-unit Dutch Auction
- Multi-unit English Auction
- Ausubel Auction

Multi-unit Sealed-bid Auction - A review

- (i) Buyers submit sealed bid vectors, with each element represents the marginal price;
- (ii) Seller allocates items to K elements with highest marginal prices in all bid vectors;
- (iii) Pricing mechanisms are very different.

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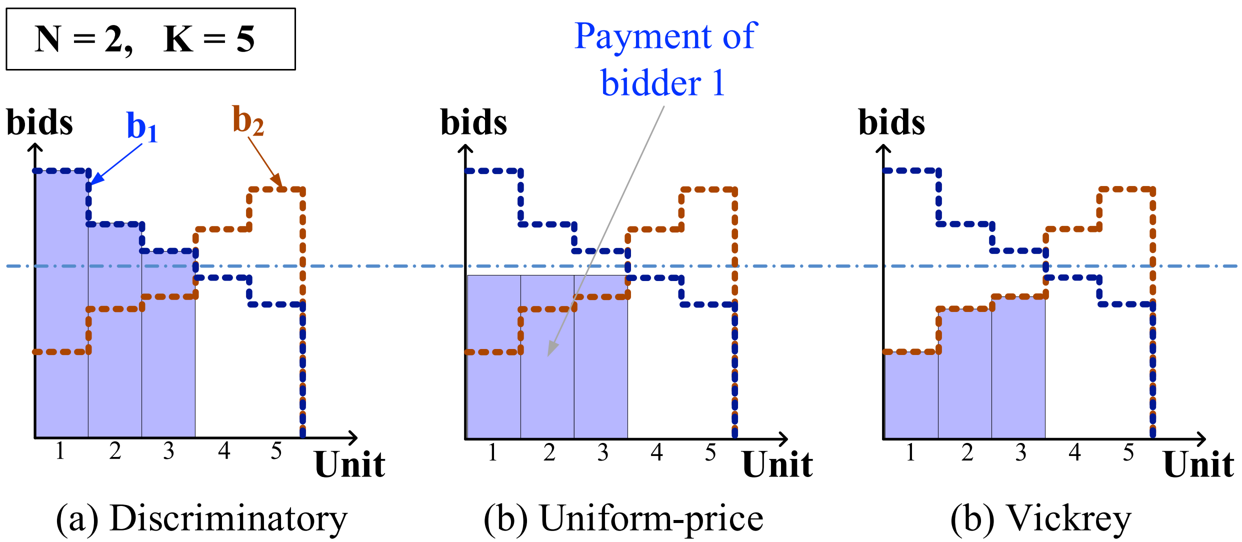


Pricing Mechanism in Multi-unit Sealed-bid auctions

Multi-unit English Auction

- The auctioneer calls an initial low price and then gradually raises the price until the total demand is equal to K.
- $\mathbf{V}=\mathrm{sort}(\mathbf{v}_1\ \mathbf{v}_2\ \ldots\ \mathbf{v}_N)$, i.e., merger all valuation vectors and sort the merger-vector in descending order.
- The demand is equal to K as soon as the price exceeds the (K+1)-th element in $\mathbf{V}$. The first K elements in $\mathbf{V}$ win the items with a price equal to the (K+1)-th element.

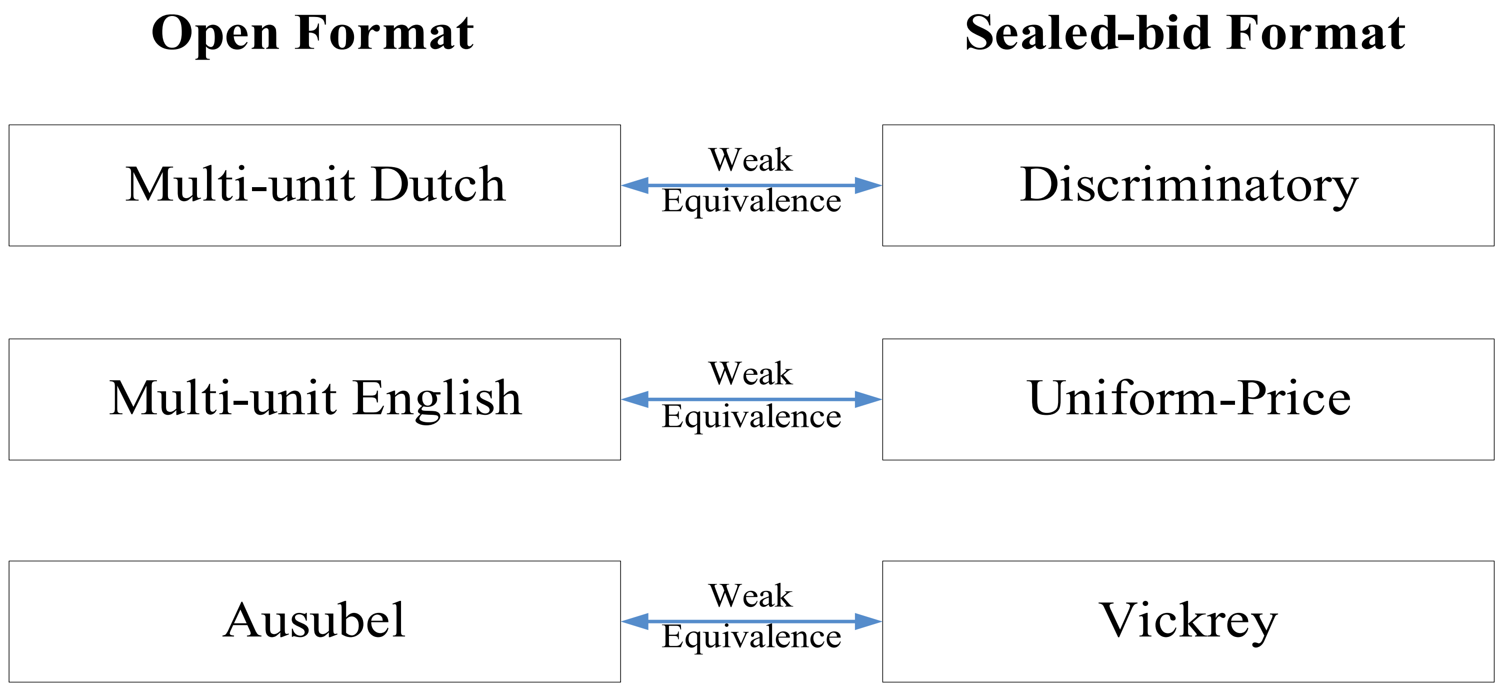
Yields the same outcome as Uniform-price sealed-bid auction !

Multi-unit Dutch Auction

- The auctioneer calls an initial high price and then gradually decreases the price until the total demand is equal to K.
- Each bidder must strategically decide at which price he is going to accept an extra item. Tradeoff : accepting a high price implies high win-probability but low utility.

Yields the same outcome as Discriminatory sealed-bid auction !

Outcome Equivalence



Applications

Example 1 - Power Allocation

- 1 base station (BS) provides services for N mobile users (MSs) operating in Orthogonal channels
- BS's total transmission power is bounded in Pmax
- Objective: maximize the sum rates of all MSs

$$\max_{\{p_i\}} \; R = \sum_i \log(1+h_i * p_i)$$

$$\text{s.t. } \sum_i p_i \leq P_{max}$$

Example 1 - Power Allocation

- Auction Based Power Allocation
- Details

Example 2 - Assignment Problem

- M primary spectrum owners (POs) each owning M_i idle channels;
- N secondary users (SUs) each desires of one channel;
- Each channel can only be used by one SU;
- Revenue of SU j using a channel of PO i is v_{ij};
- Objective: find a feasible assignment maximizing the social welfare

$$\max_{\{r_{ij}\}_{MxN}} \quad R = \sum_i \sum_j r_{ij} v_{ij}$$

$$\text{s. t.} \quad r_{ij} \in \{0,1\} \quad \forall i, \forall j$$

$$\sum_j r_{ij} \le M_i, \forall i, \text{ and } \sum_i r_{ij} \le 1, \forall j$$

Example 2 - Assignment Problem

- Auction Based Optimal Assignment
- Details

Conclusion & Future

Conclusion

- We discussed dynamic auction mechanisms for different market models.
- We discussed two illustrations of applying dynamic auction in wireless networks.

Important Surveys

- Richard Engelbrecht-Wiggans 1980
  - Auctions and Bidding Models: A Survey
- Robert Stark and Michael Rothkopf 1979
  - Competitive Bidding: A Comprehensive Bibliography
- Paul Milgrom (1985, 1986)
  - Auction Theory
- Robert Wilson (1987).
  - Auction Theory

Auction Designs in Wireless Networks

- Sharing Auction (Divisible Goods) [Jianwei Huang-08]
  - 1->N, Sealed-bid Format, Centralized Control
  - Power/SNR Allocation: fairness & efficiency
- eBay-like Spectrum Auction [Haitao Zheng-08]
  - 1->N, Sealed-bid Format, Centralized Control
  - Spectrum Reuse, Revenue Maximizing
- Truthful Double Auction [Haitao Zheng-09]
  - M->N, Sealed-bid Format, Centralized Control
  - Spectrum Reuse, Revenue Maximizing
- MAP: Dynamic Auction [Lin Gao-10]
  - M->N, Open Format, Decentralized
  - No Spectrum Reuse, Social Spectrum Efficiency Maximizing

Thanks