Online Mechanism Design (I) —— Basics

Lin Gao

Network Communications and Economics Lab (NCEL)

Department of Information Engineering

The Chinese University of Hong Kong

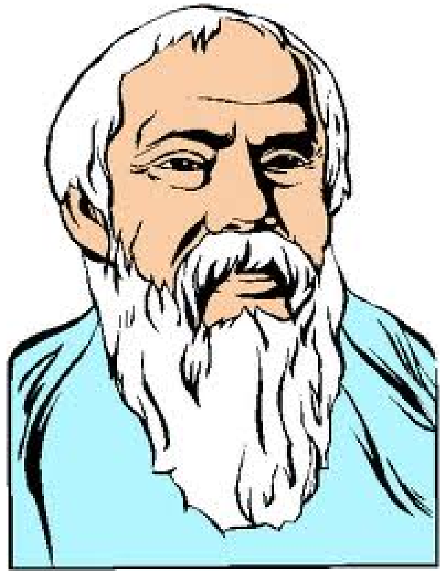
Outline

- Examples of Online Scheduling Problems
  - The Socrates' Problem
  - The Persian Princess' Marriage Problem
- Online Mechanism Design
  - Mechanism Design
  - Roadmap to Online Mechanism Design
  - Online auction mechanism
- Conclusion

The Socrates' Problem

What is love?

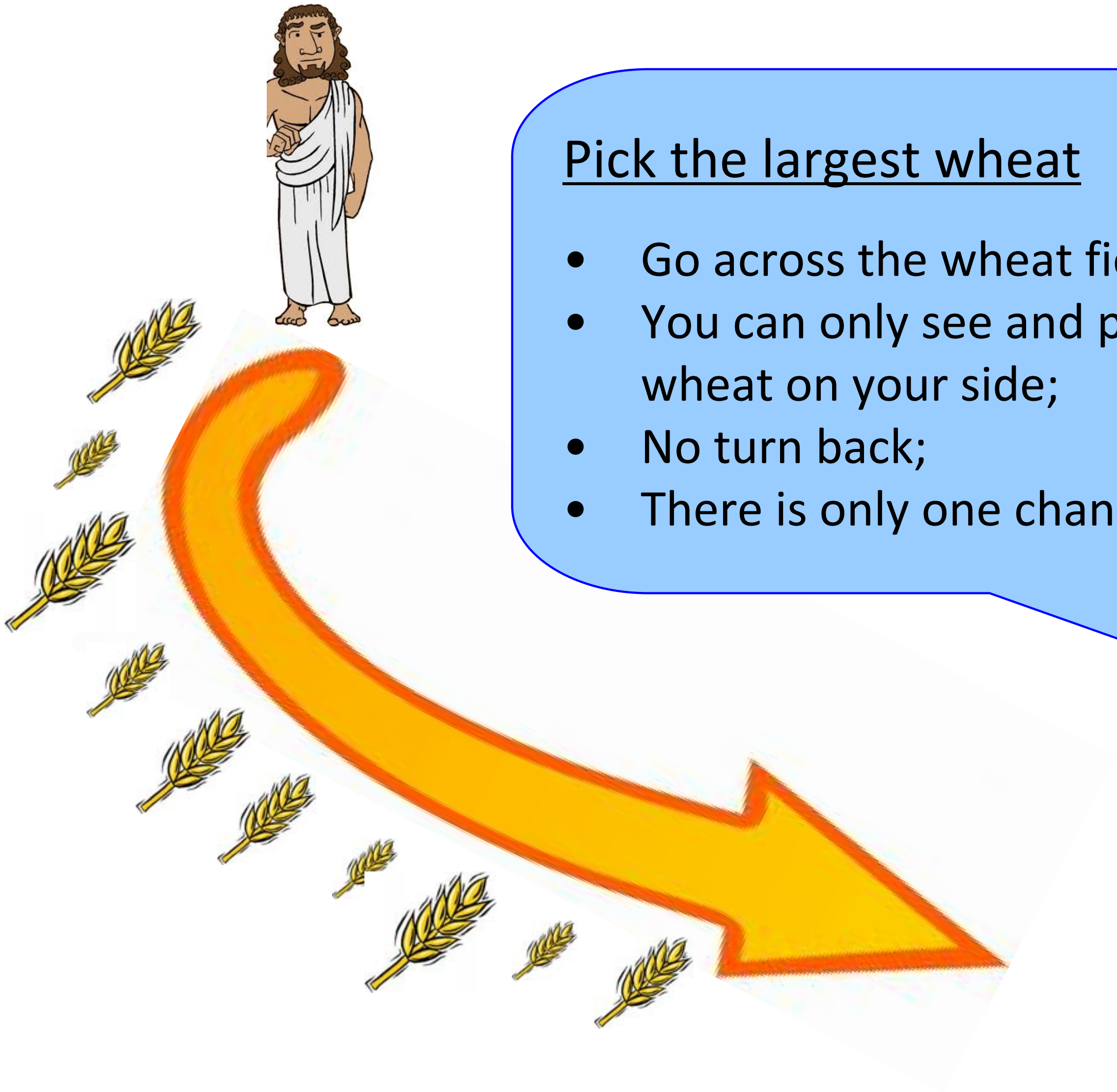
Love is just like picking the largest wheat in a field …



Socrates (469 BC – 339 BC)

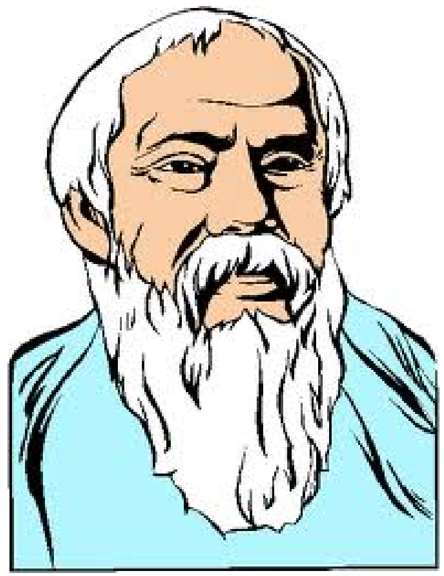
Plato (424 BC – 348 BC)

The Socrates' Problem



Pick the largest wheat

- Go across the wheat field;
- You can only see and pick the wheat on your side;
- No turn back;
- There is only one chance.



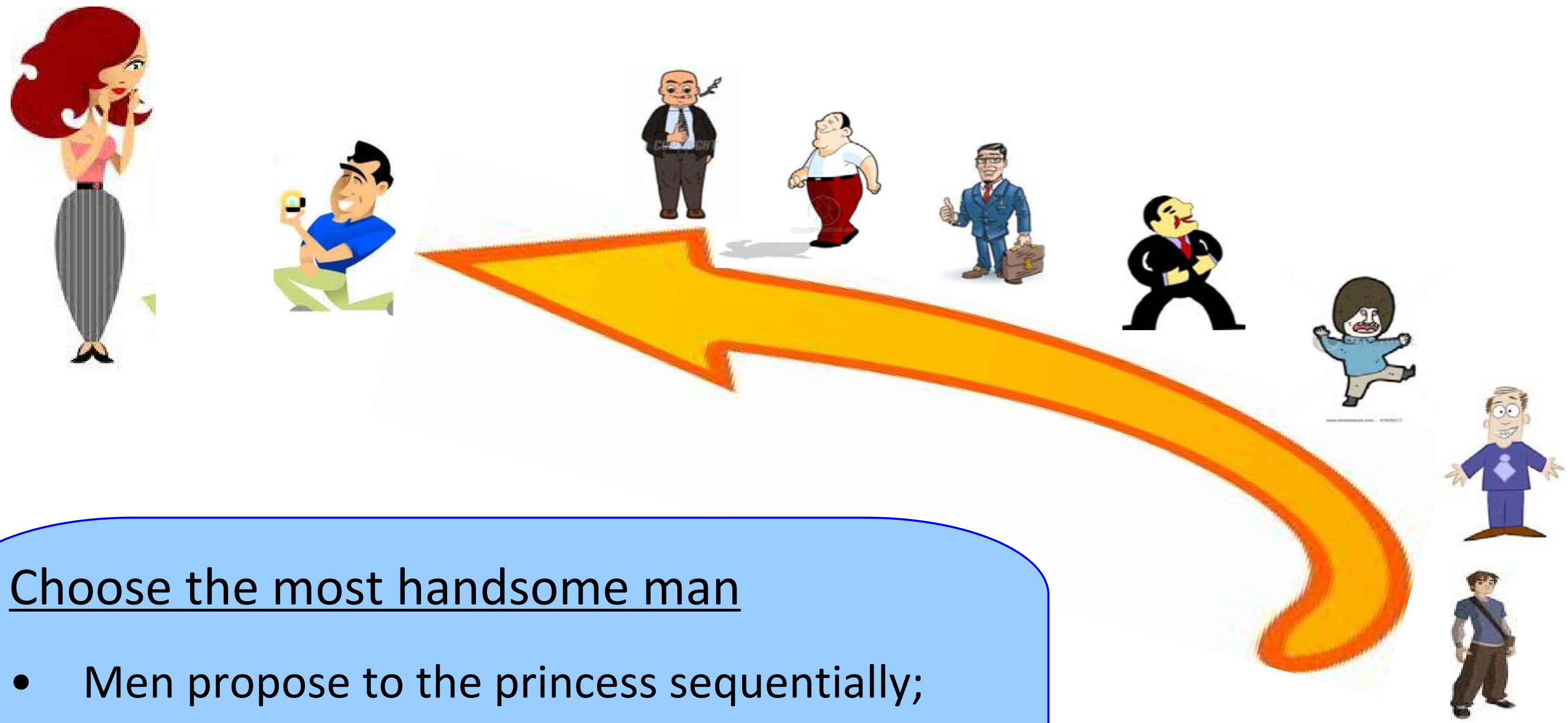
Socrates (469 BC – 339 BC)

The Socrates' Problem

Persian Princess' Marriage Problem

I will choose a husband for my princess from a given set of candidates …

Persian Princess' Marriage Problem



Choose the most handsome man

- Men propose to the princess sequentially;
- Can only see and accept the proposing man;
- Cannot accept a man who has been refused previously (No turn back);
- Once accepting, game over (One chance).

Persian Princess' Marriage Problem

Is this guy the most handsome one?

Online Scheduling Problem

- Basic model
  - There is one item to be allocated to one potential demander;
  - A set of N demanders request sequentially with a random order;
  - The demanders have different valuations V_n for the item.
- Assumption
  - i.i.d. valuation V_n
- Objective
  - Allocate the item to the highest valuation demander.

Online Scheduling Problem

- Key features
  - No future information
    - Can only see the wheat on the side (the proposing man);
  - Opportunity is fleeting (Real-time decision)
    - Can only pick the wheat on the side (accept the proposing man), and no turn back;
  - No withdrawing (No preemption)
    - One chance: Once picking (accepting), game over.
- More features
  - No a-priori stochastic information

Online Scheduling Problem

- Objective
  - Allocate the item to the highest valuation demander.
- We cannot guarantee to achieve this objective with 100%;
- Or, we can only achieve this objective with a certain probability.

What is the largest probability we can achieve?

Can we find a scheduling rule to achieve this probability?

Online Scheduling Problem

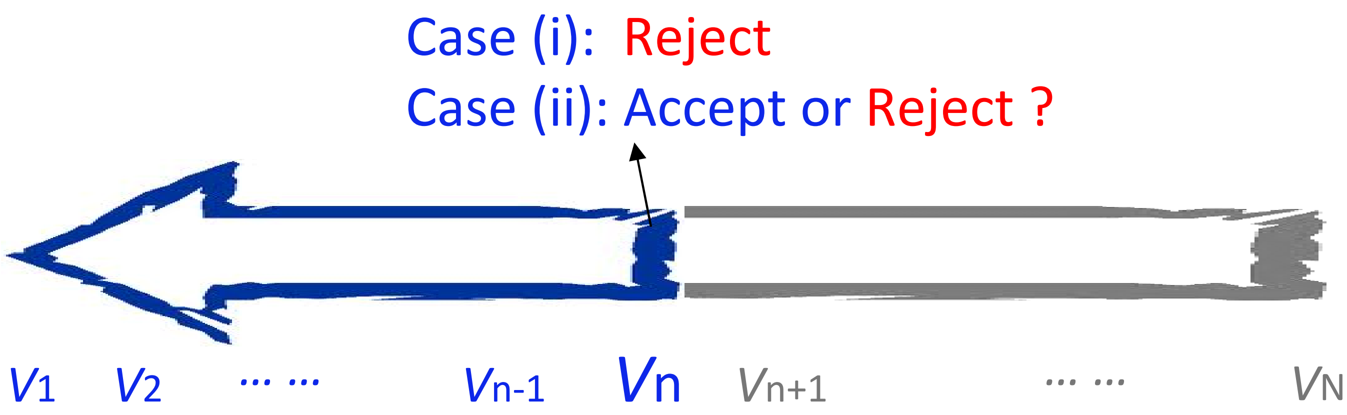
- The largest probability

The largest probability (of allocating the item to the highest valuation demander) is:

$$1/e = 1/2.718 = 36.79\%$$

Online Scheduling Problem

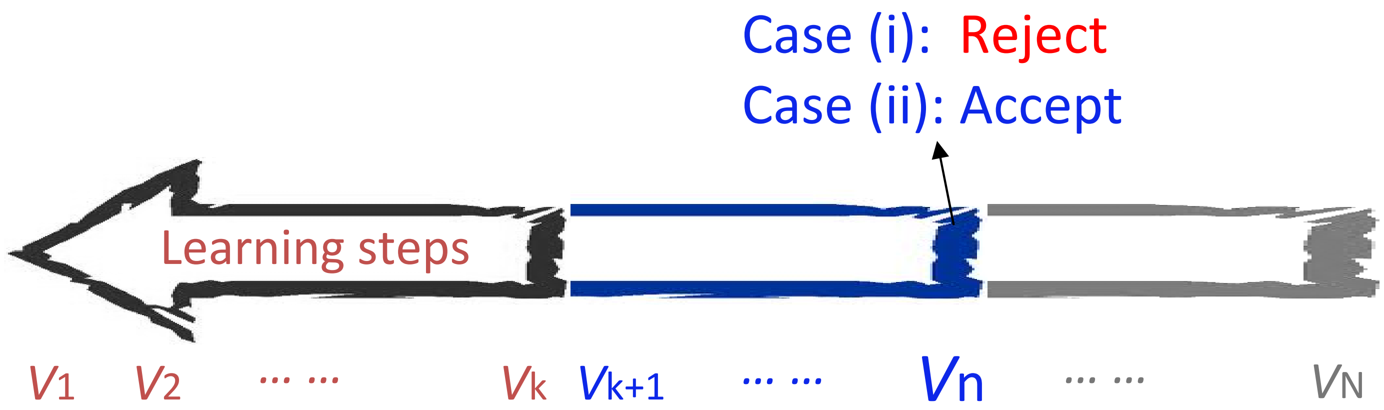
- Optimal scheduling rule
  - Suppose w.l.o.g demander n requests at the step n.
  - At step n: demander n with valuation V_n
    - Case (i): there exists an $V_{i,\, i<n}$ such that exists $V_i > V_n$
    - Case (ii): V_n is the largest valuation among all requested n demander



  - From which step (called critical step) on, a demander n satisfying case (ii) will be accepted?

Online Scheduling Problem

- Optimal scheduling rule
  - Suppose a critical step *k*
    - Before step *k*: Reject all demanders *1,2,…k*
    - From step *k+1*: Accept the first demander satisfying case (ii)



What is the optimal critical step *k*?

Online Scheduling Problem

- Optimal scheduling rule
  - Given a critical step k
    - (1) Prob = 0, If the largest valuation is within $V_{i,\ i=1,\dots,k}$
    - (2) Prob = 1, If the largest valuation is V_{k+1}
    - (3) Prob = k/(k+1), If the largest valuation is V_{k+2}
      - The largest valuation among all previous k+1 demanders is within $V_{i,\ i=1,\dots,k}$
    - (4) Prob = k/(k+2), If the largest valuation is V_{k+3}
      - The largest valuation among all previous k+2 demanders is within $V_{i,\ i=1,\dots,k}$
    - … …
    - Finally, Prob = k/(N-1), If the largest valuation is V_N
      - The largest valuation among all previous N-1 demanders is within $V_{i,\ i=1,\dots,k}$

$$P(k) = 1/N * [k/k + k/(k+1) + k/(k+2) + \dots + k/(N-1)]$$
$$= k/N * [1/k + 1/(k+1) + 1/(k+2) + \dots + 1/(N-1)]$$

Online Scheduling Problem

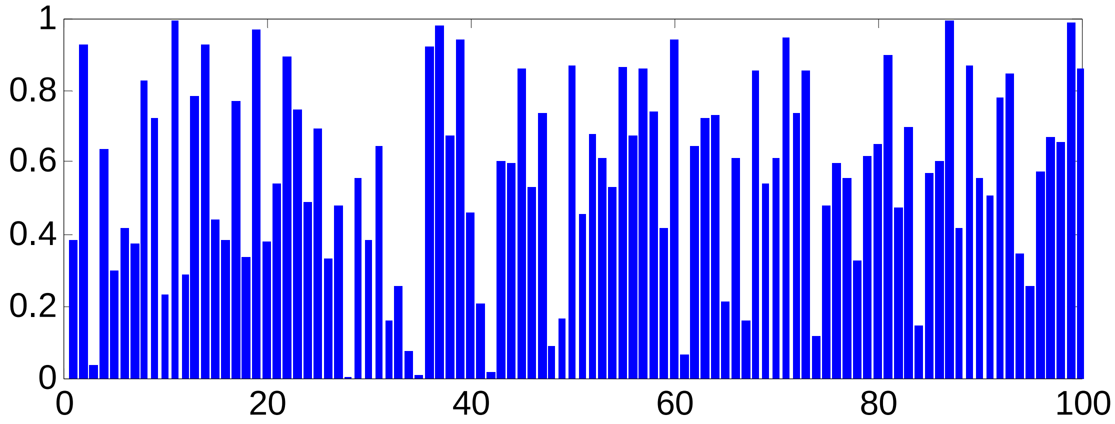
- Optimal scheduling rule
  - The optimal critical step k^*

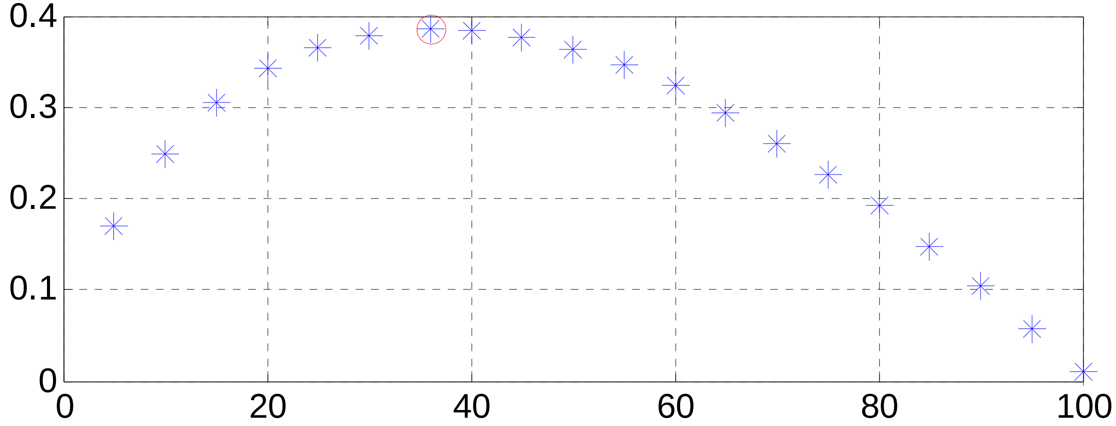
$$k^* = \arg\max_{k} P(k)$$

$$k^* = N/e = N \times 0.3679$$
$$P(k^*) = 1/e = 36.79\%$$

Online Scheduling Problem

- Test the probability: 36.79%





Online Scheduling Problem

- Discussion

  If there is no …

  - (1) No future information
  - (2) Opportunity is fleeting (Real-time decision)
  - (3) No withdrawing (No preemption)
  - (4) No a-priori stochastic information

Online Scheduling Problem

- Further discussion

  In wireless network, online scheduling problems may be …

  - (1) Objective: Expected overall valuation (social welfare)
  - (2) A-priori stochastic information: {Part, Complete}
  - (3) No future information
  - (4) Opportunity is fleeting (Real-time decision)
  - (5) Withdraw (Preemption): {Yes, No}

Outline

- Examples of Online Scheduling Problems
  - The Socrates' Problem
  - The Persian Princess' Marriage Problem
- Online Mechanism Design
  - Mechanism Design
  - Roadmap to Online Mechanism Design
  - Online auction mechanism
- Conclusion

Why Mechanism Design

- An implied assumption in the scheduling problem
  - The demander's valuation is observable at the time the demander requests.
    - E.g., Plato can see the wheat on his side at every step;
    - E.g., Princess can see the proposing man at every step.
- In many cases, this assumption may be too strong.
  - The demander's valuation cannot be observed by others.

    (also called private information, information asymmetry)
- Purpose of mechanism design
  - To achieve a desirable scheduling or allocation under information asymmetry.

Why Mechanism Design

- Information Asymmetry in Persian Princess' Marriage Problem

Why Mechanism Design

- Incentive Compatibility
  - An incentive compatible mechanism provides the incentive for demanders truthfully report their private information.
- Efficiency – Social Welfare Maximization
  - An efficient mechanism maximizes the expected social welfare (the total valuation of all scheduled users).
- Optimality – Revenue Maximization
  - An optimal mechanism maximizes the expected revenue (the total payment of all scheduled users) of the mechanism designer.

Mechanism Design vs Scheduling

Mechanism Design

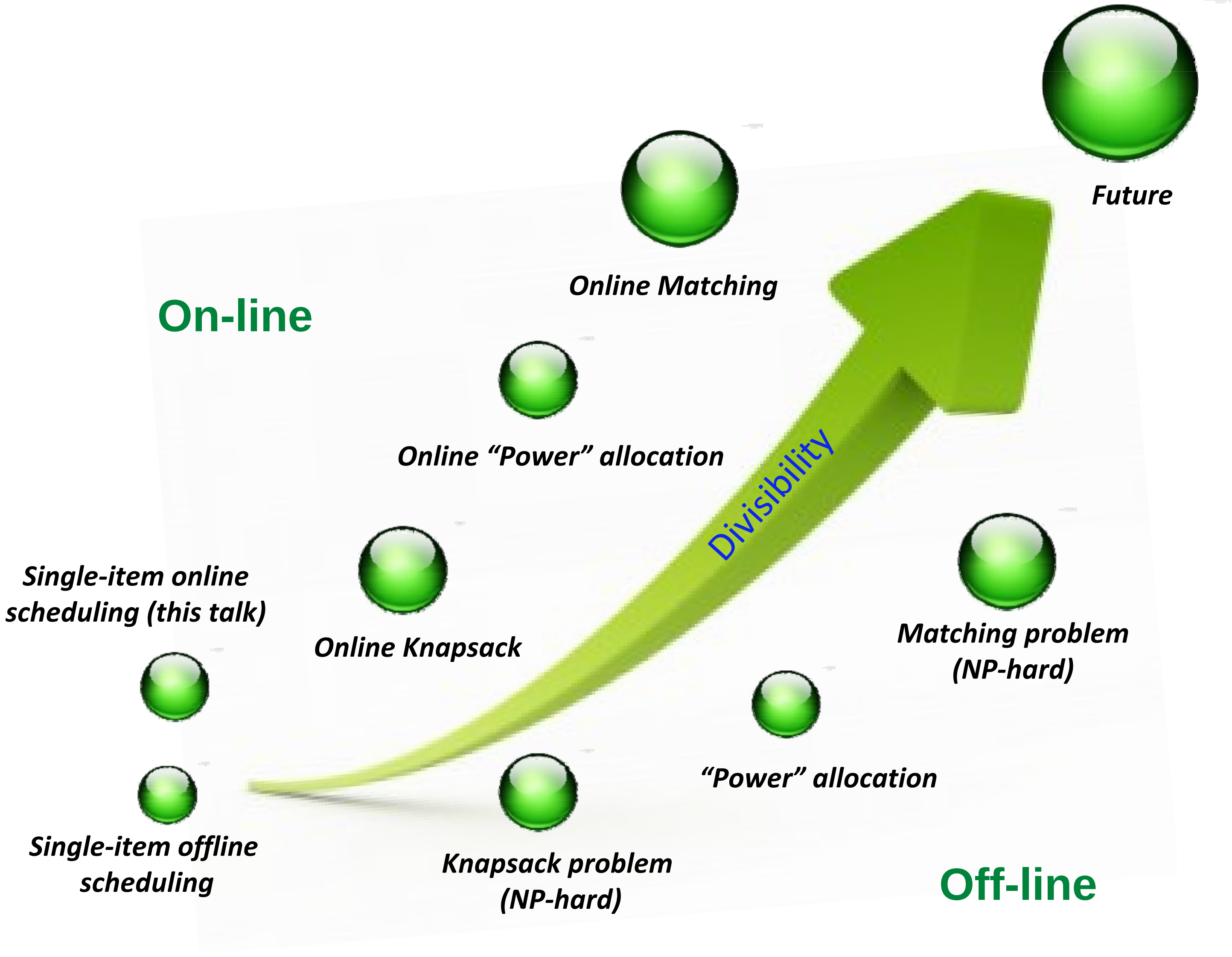
Scheduling Rule

Achieve desirable scheduling or allocation based on the users' truthful report.

Payment Rule

Provide incentive for users truthfully reporting their private information

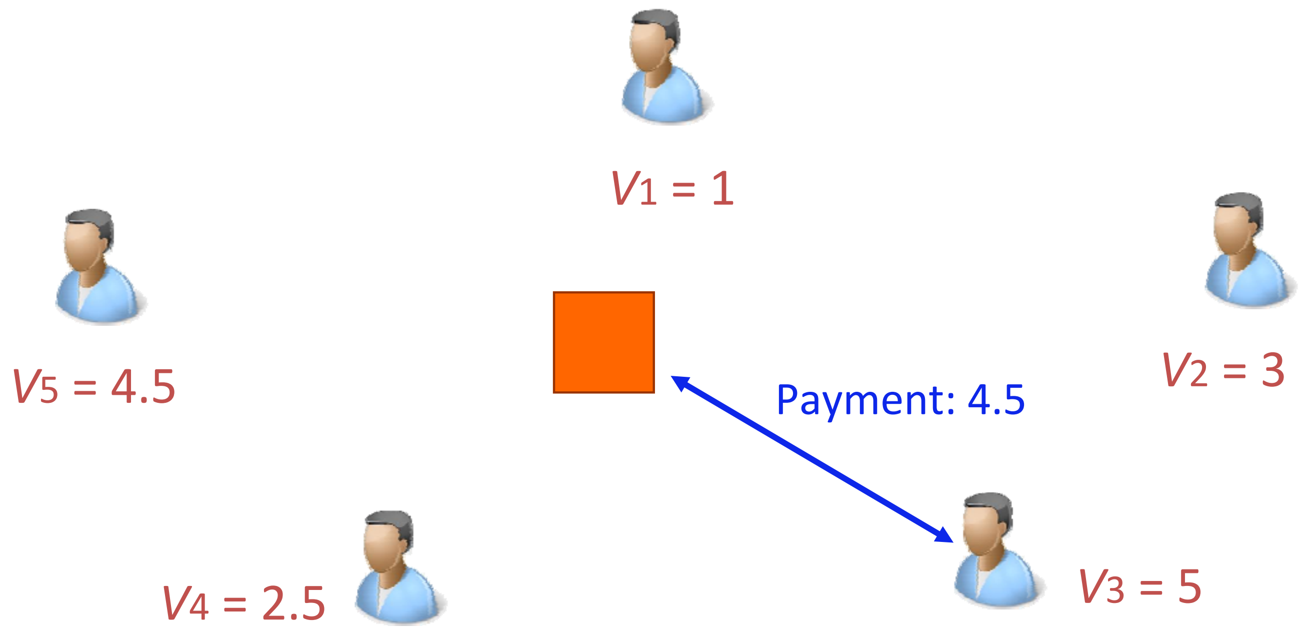
Roadmap



Future
Online Matching
On-line
Online "Power" allocation
Divisibility
Single-item online scheduling (this talk)
Online Knapsack
Matching problem (NP-hard)
"Power" allocation
Single-item offline scheduling
Knapsack problem (NP-hard)
Off-line

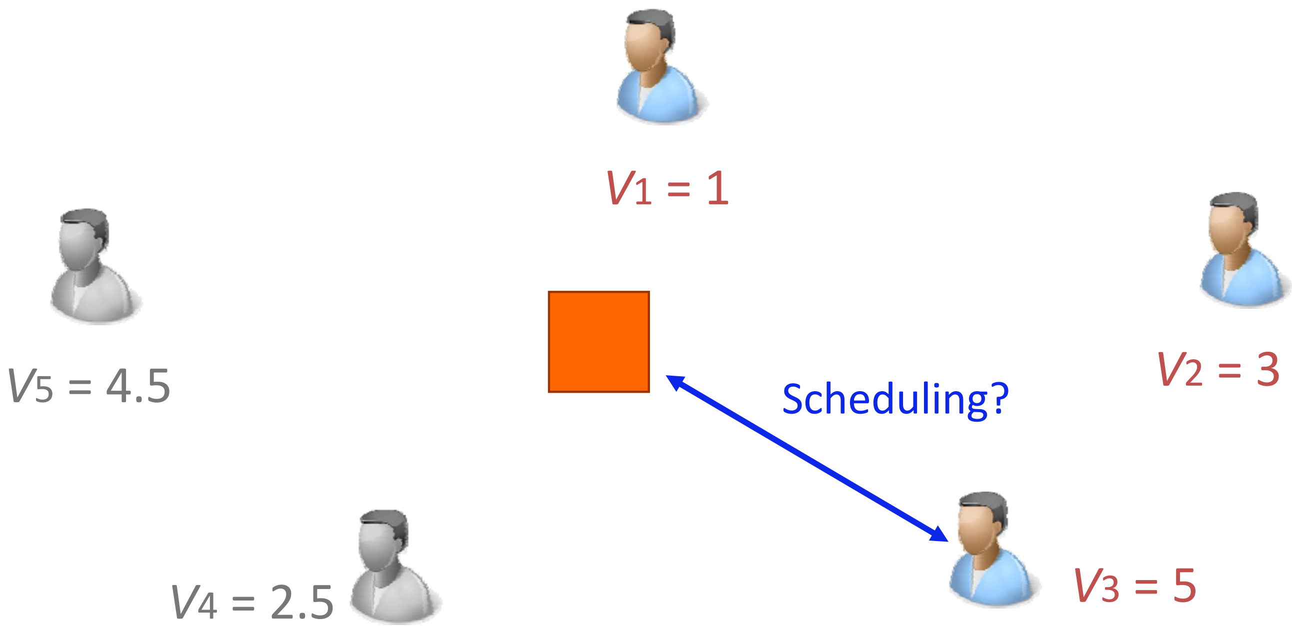
Roadmap

- Single-item Offline Scheduling
  - Maximizing the social welfare
    – Simple searching
- Mechanism Design
  - Classic VCG Mechanism, e.g., 2nd-Price Auction



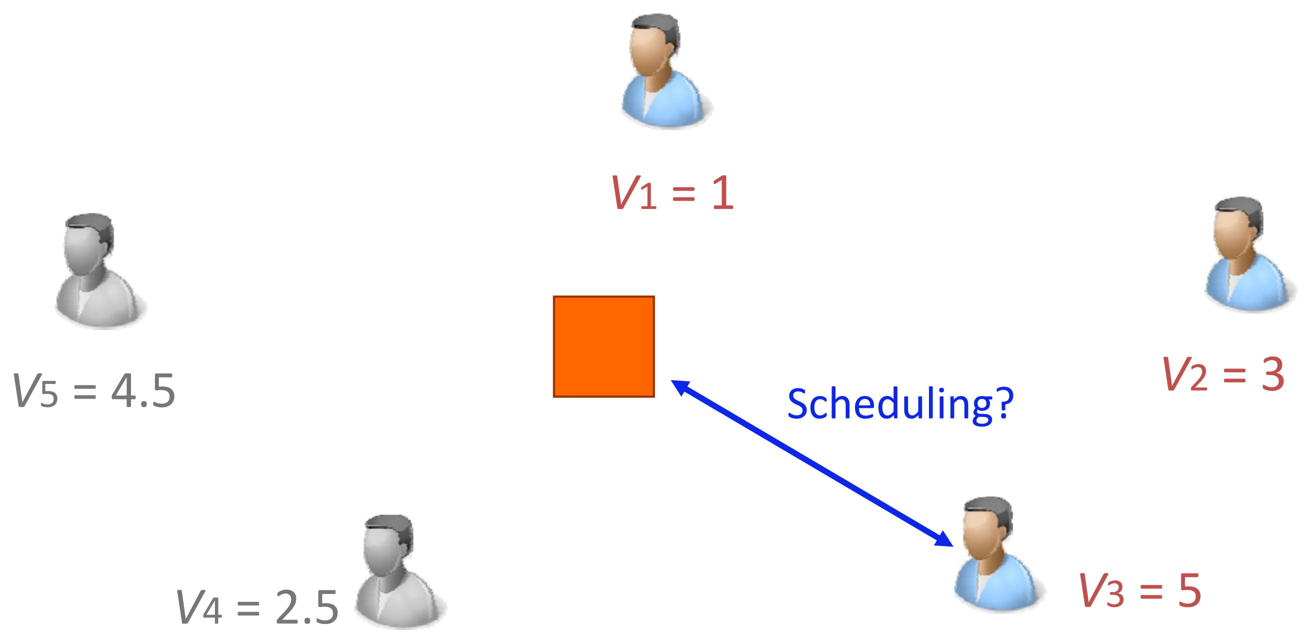
Roadmap

- Single-item Online Scheduling
  - Maximizing the successful probability
    – Learning-before-scheduling
- Mechanism Design
  - Question ?



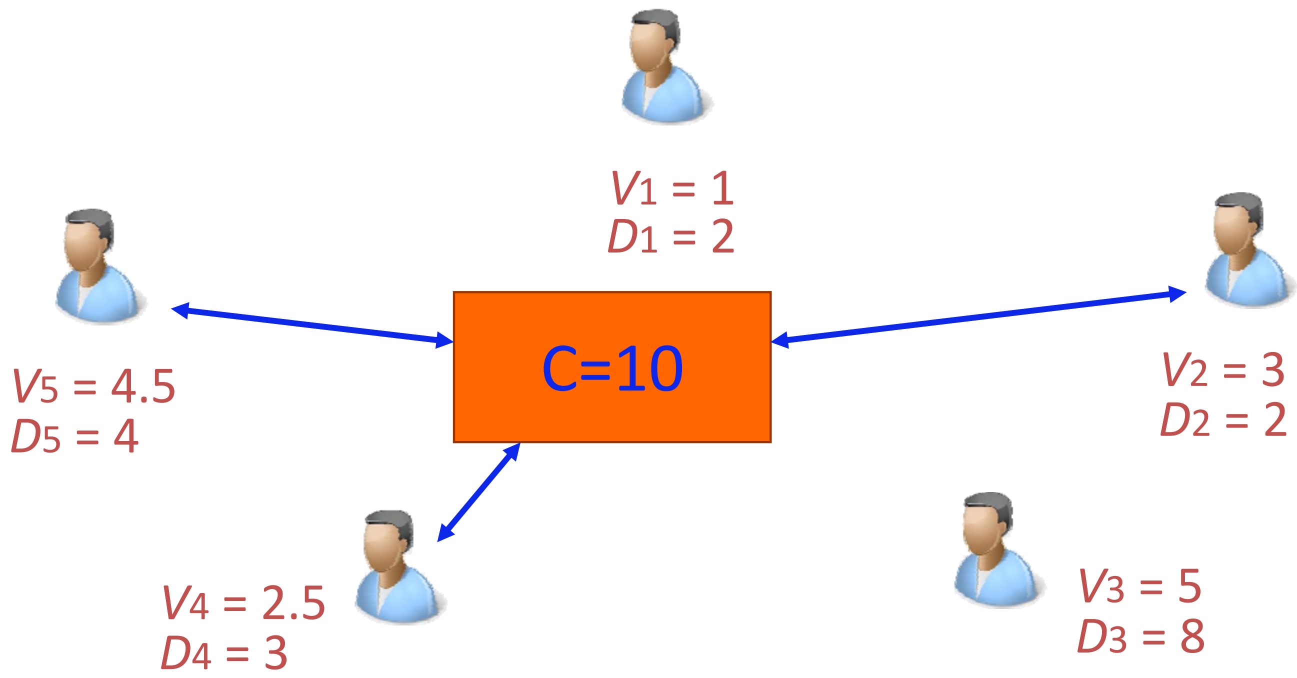
Roadmap

- Single-item Online Scheduling
  - Maximizing the social welfare
    – Greedy Scheduling
- Mechanism Design
  - Question ?



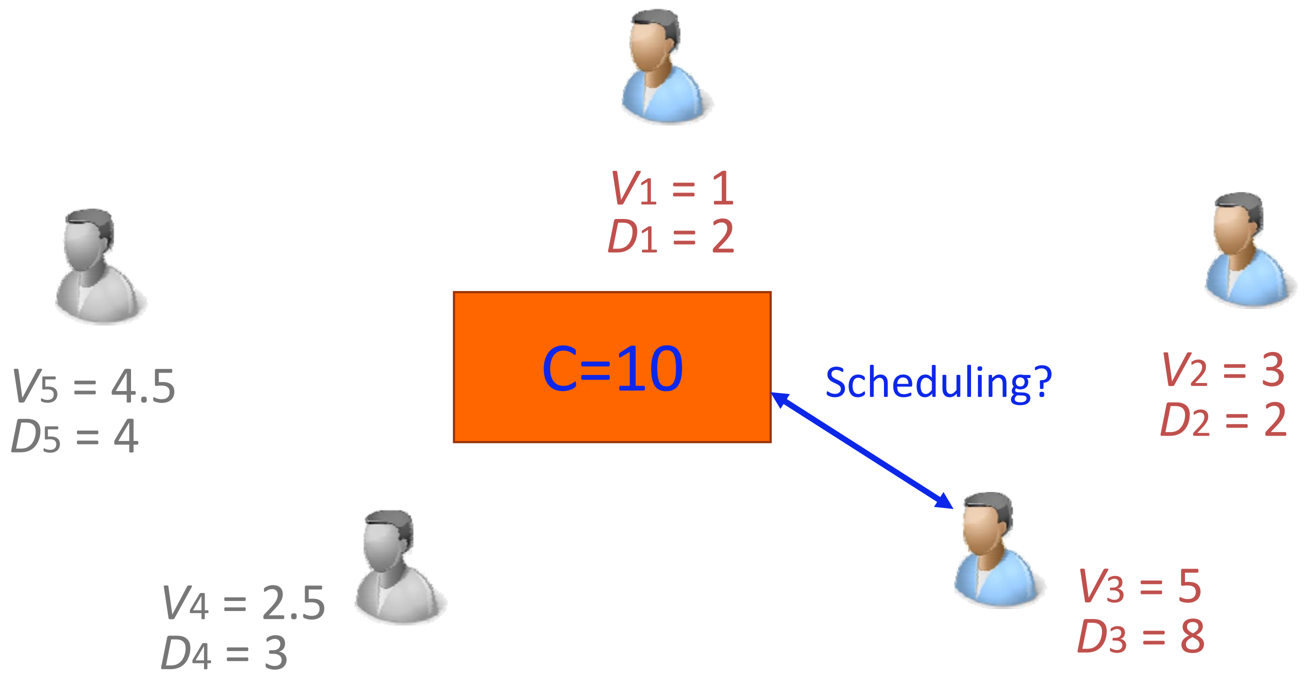
Roadmap

- Knapsack Problem
  - Maximizing the social welfare s.t. *sum(Di)* ⩽ *C* – NP-hard
- Mechanism Design
  - Classic VCG Mechanism



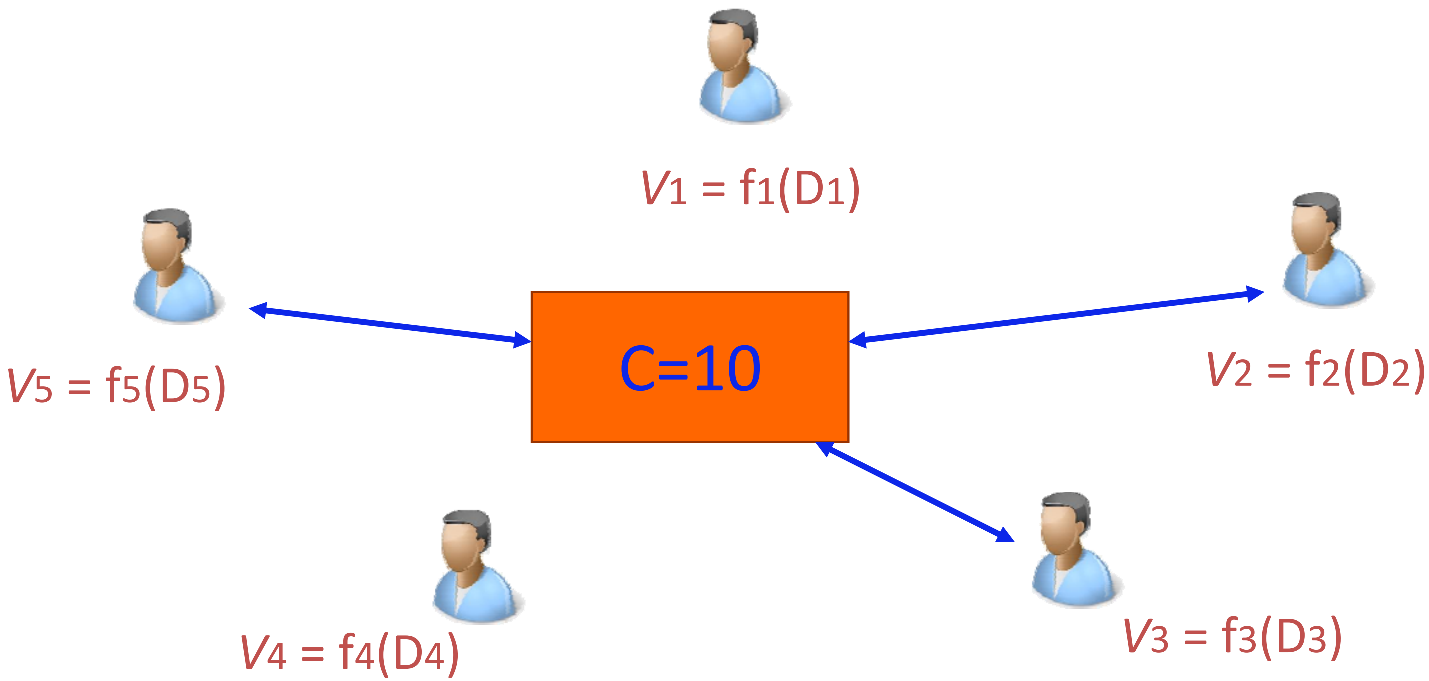
Roadmap

- Online Knapsack Problem
  - Maximizing the social welfare s.t. *sum(Di)* $\leqslant$ *C*
  - – [Deniz Dizdar 2009]
- Mechanism Design
  - – [Deniz Dizdar 2009]



Roadmap

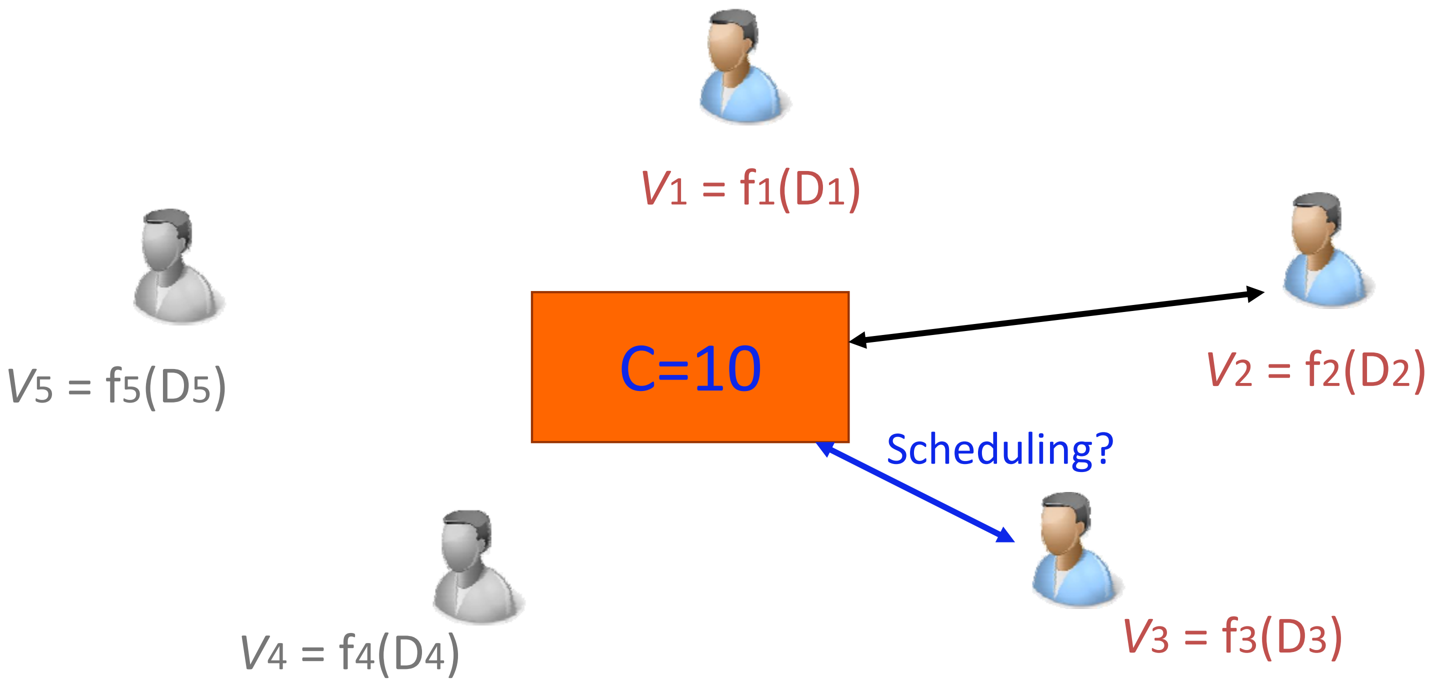
- "Power" Allocation Problem
  - Maximizing the social welfare s.t. *sum(Di)* $\leqslant$ *C*
    – Water-filling
- Mechanism Design
  - Question ? [Hint: VCG]



Roadmap

- Online "Power" Allocation
  - Maximizing the social welfare s.t. *sum(Di)* $\leqslant$ *C*

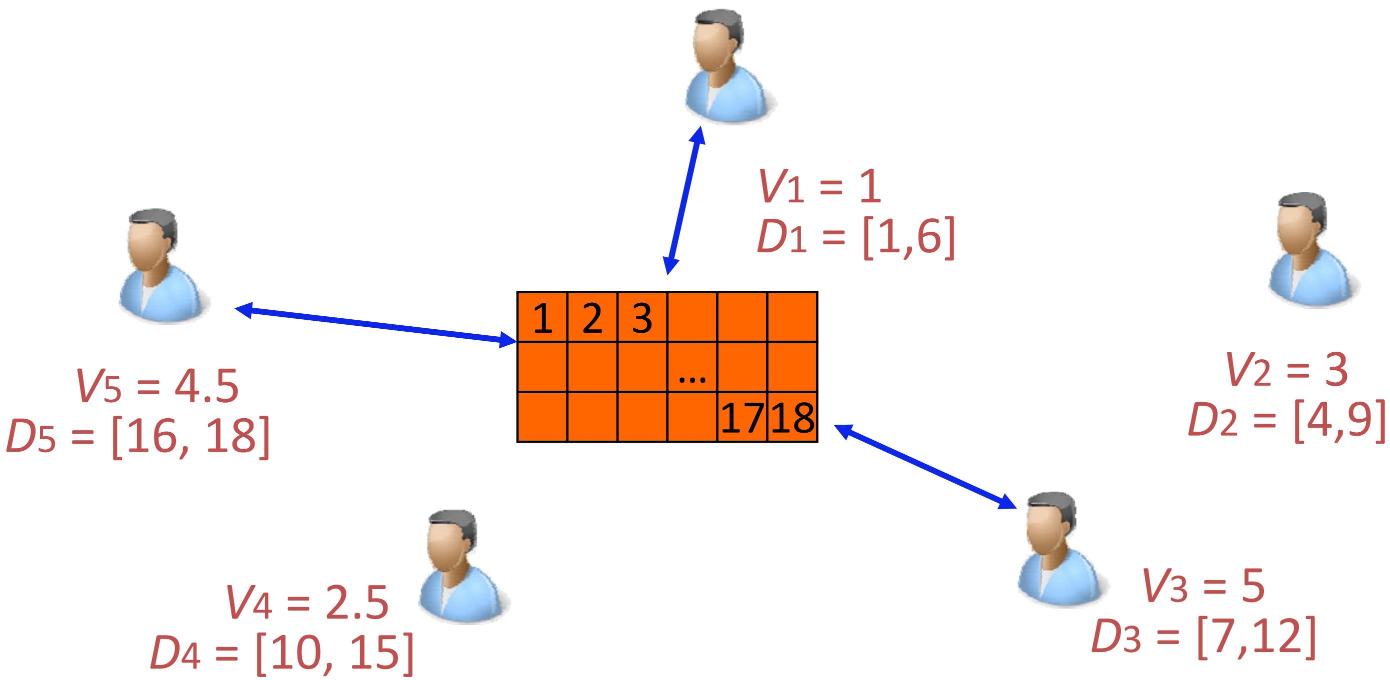
    – [Ishai Menache 2012]
- Mechanism Design
  - – [Ishai Menache 2012]



Roadmap

- Matching Problem
  - Maximizing the social welfare

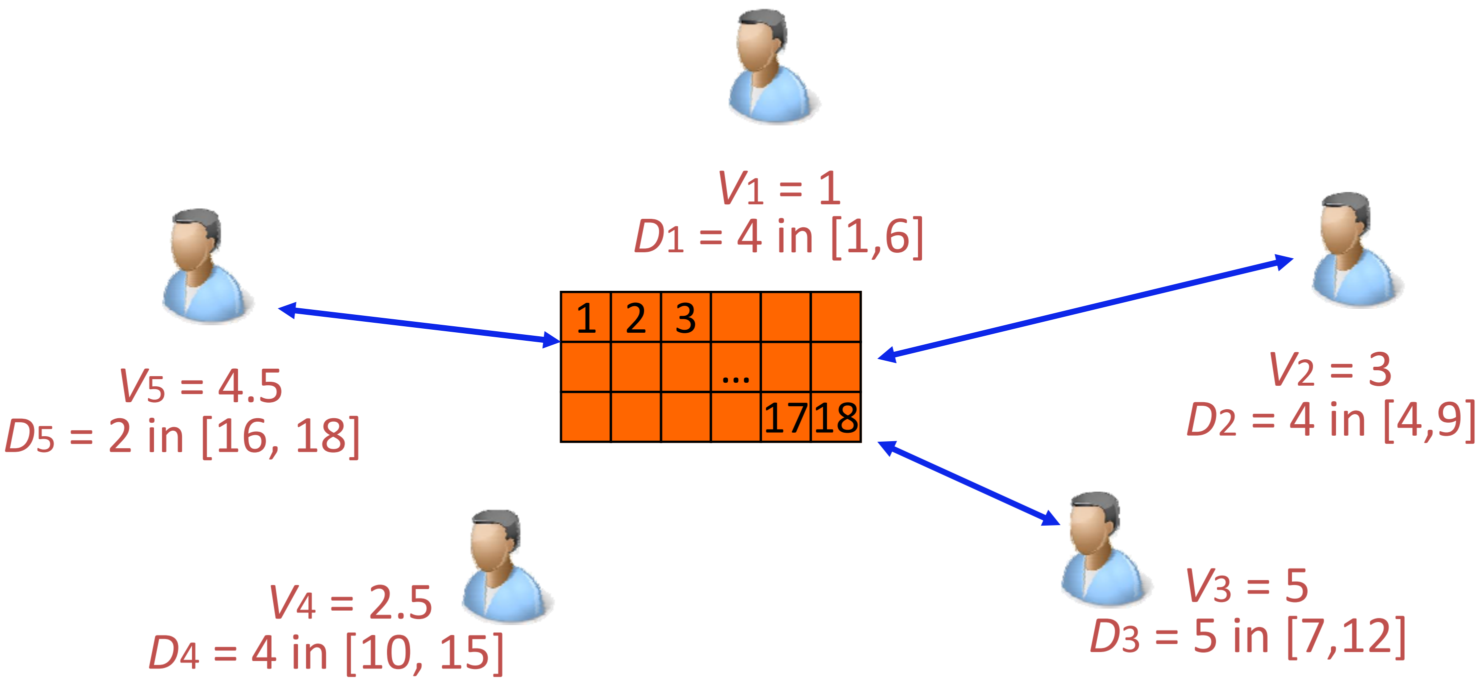
    – MWIS (NP-hard)
- Mechanism Design
  - – Classic VCG Mechanism



Roadmap

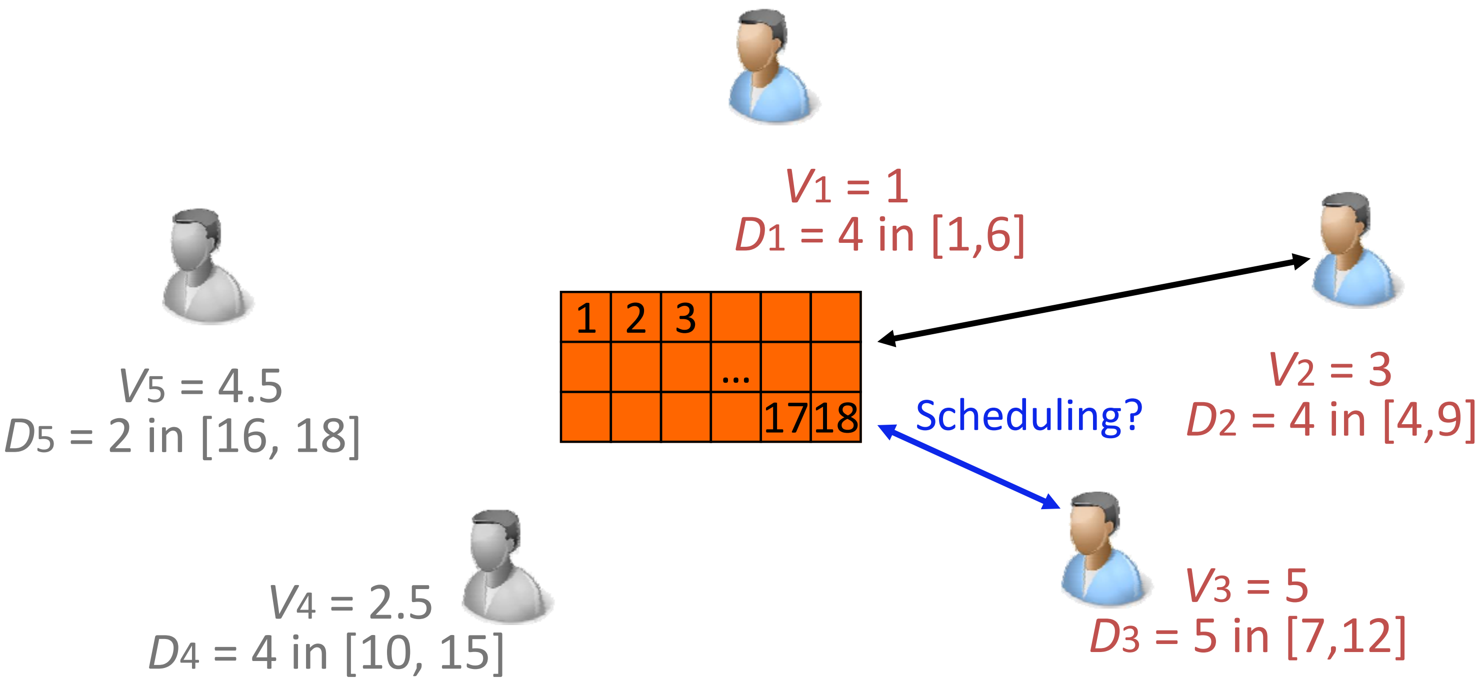
- Generalized Matching Problem
  - Maximizing the social welfare

    – MWIS (NP-hard)
- Mechanism Design
  - – Classic VCG Mechanism



Roadmap

- Online Matching Problem
  - Maximizing the social welfare
    – [Mohammad T. Hajiaghayi 2005 EC]
- Mechanism Design
  - – [Mohammad T. Hajiaghayi 2005 EC]

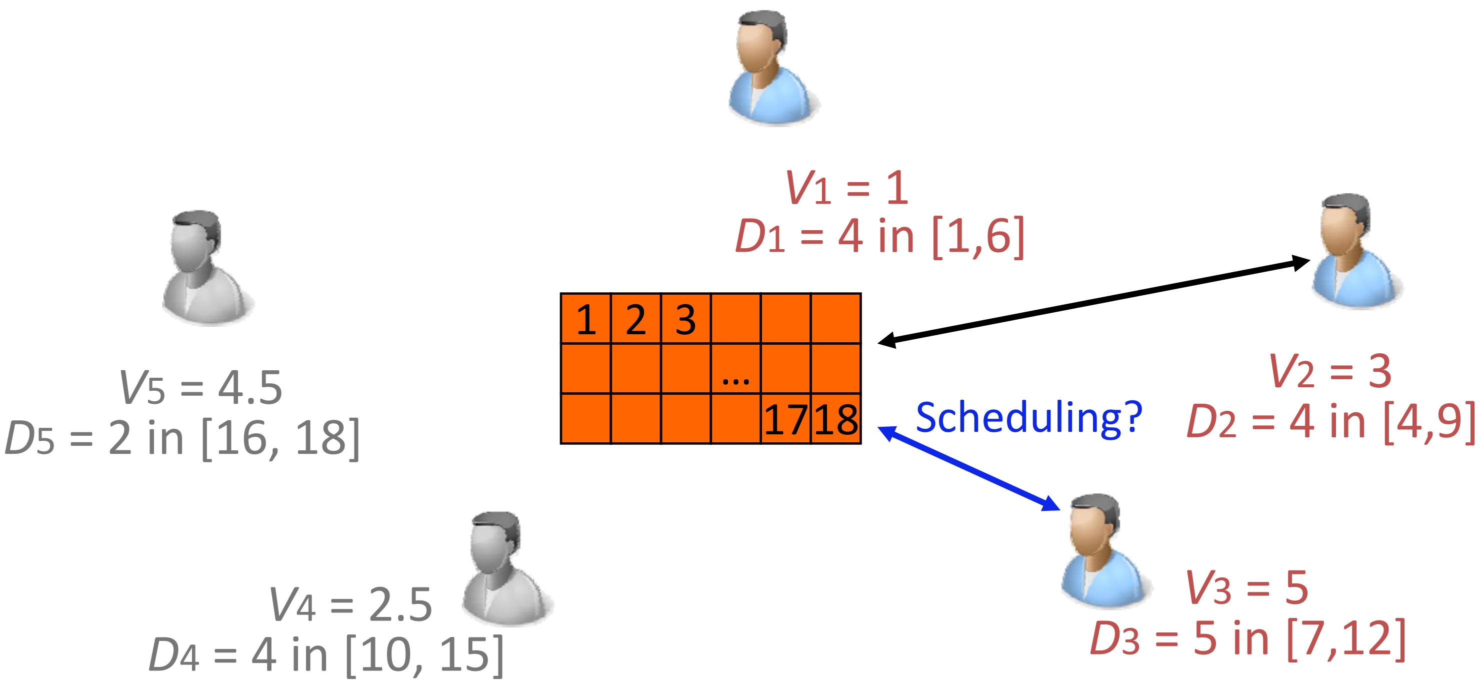


Outline

- Examples of Online Scheduling Problems
  - The Socrates' Problem
  - The Persian Princess' Marriage Problem
- Online Mechanism Design
  - Mechanism Design
  - Roadmap to Online Mechanism Design
  - **Online auction mechanism**
- Conclusion

Online Auction Design

- Mohammad T. Hajiaghayi, 2005 EC.
- The Model
  - Private information: *Vi, Di, [Ai, Bi]*



Conclusion

- Online Scheduling Problem
- Online Mechanism Design
- Future: Online Auction Design

Thank you !