Online Mechanism Design
Part II: Online Auction

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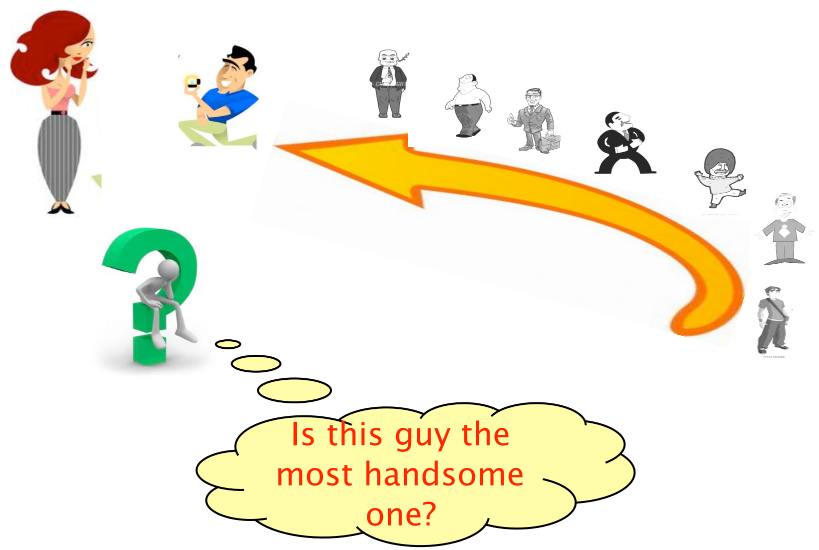
Outline

- Review of Online Mechanism Design
  - Online Scheduling Problem
  - Online Mechanism Design
  - Research Roadmap
- Online Auction Design
  - System Model
  - Online Auction Mechanism
- Conclusion

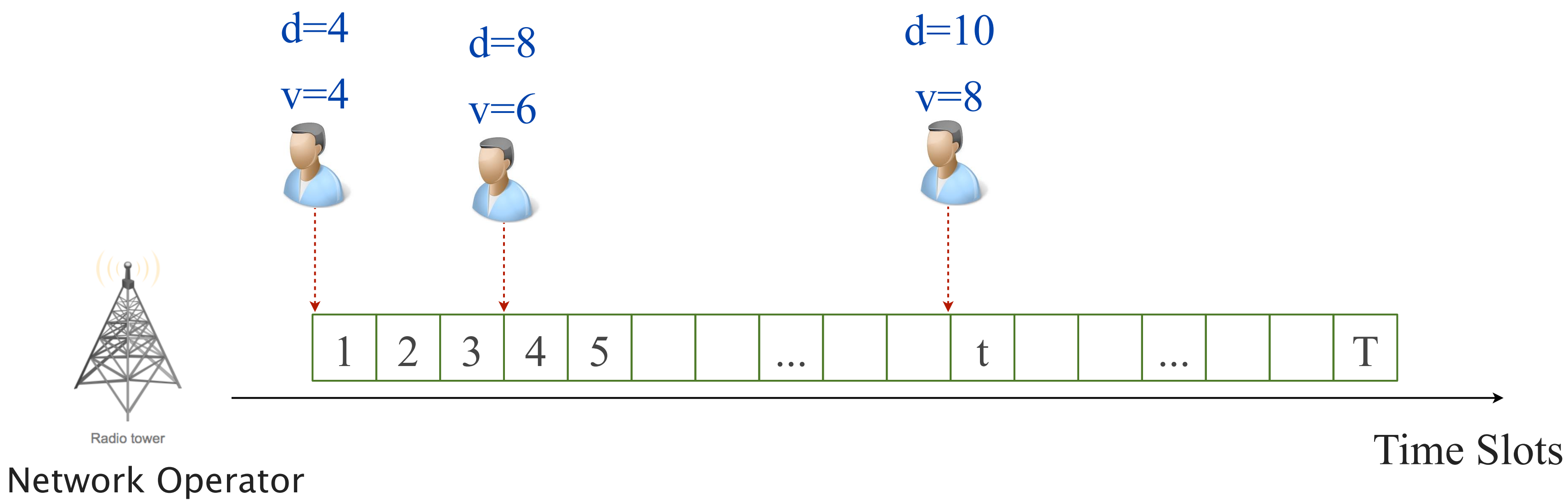
Review

- Online Scheduling Problem
  - Allocating items to sequentially arrived demanders
  - Key features
    - No future information
    - Opportunity is fleeting (Real-time decision)
    - No withdrawing (No preemption)
  - Example: Persian Princess's Marriage



Review

- Online Scheduling Problem
  - Example: Online Channel Allocation Problem
    - Mobile users arrive sequentially and randomly.
    - Channel allocation is real-time.
    - Preemption is not allowed.

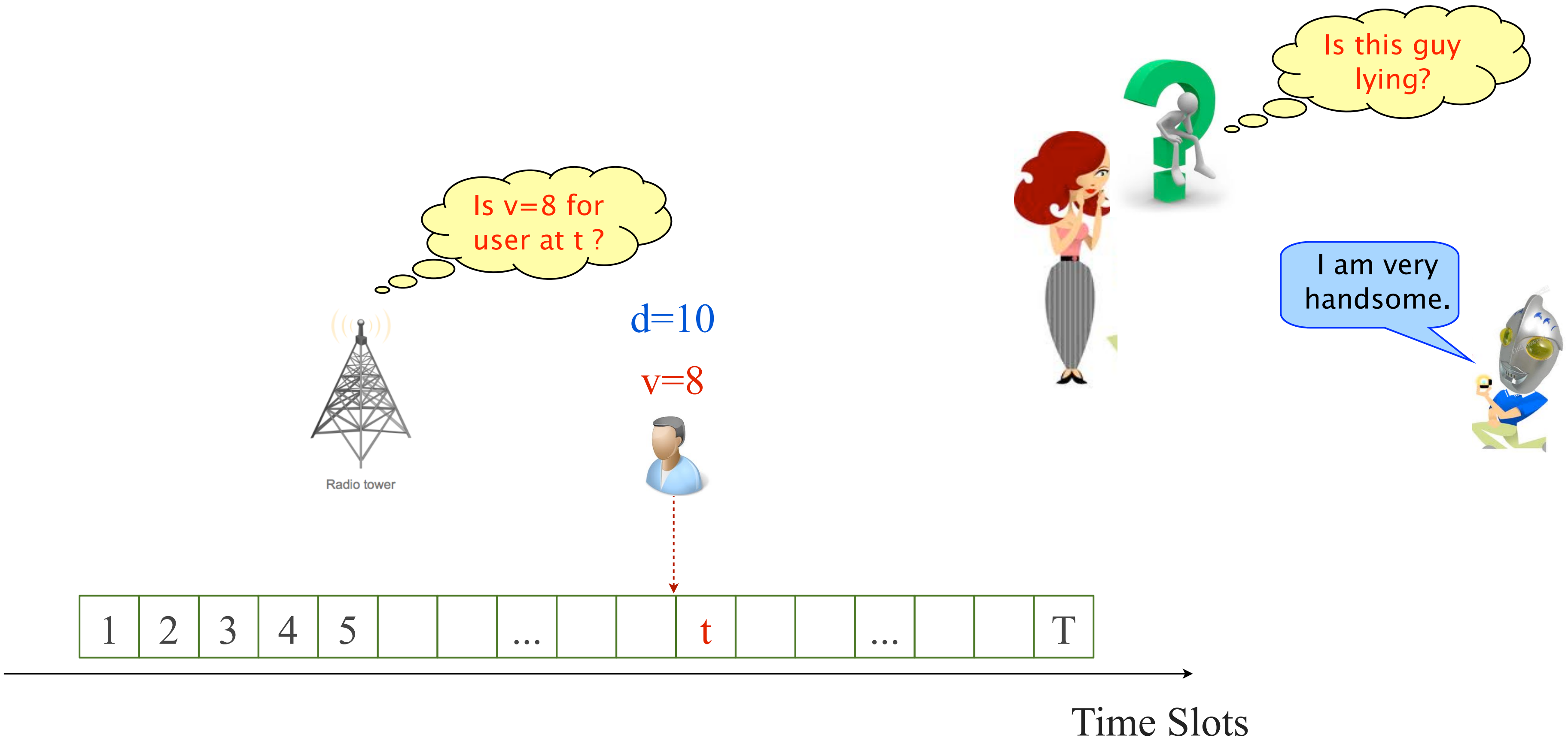


Review

- Online Scheduling Problem
  - Performance evaluation – Competitive Ratio
    - The worst case of the ratio between the performance under an online algorithm and the optimal offline performance.
    - Constant competitive ratio (e.g., 2-competitive) is expected.
  - Example: Learning-before-scheduling algorithm
    - The largest probability (of allocating the item to the highest valuation demander) is: $1/e = 1/2.718 = 36.79\%$
    - The competitive ratio is zero (i.e., not constant CR).
      - For example, when the highest valuation is within the learning stage, and all other valuations are much smaller than the highest one.

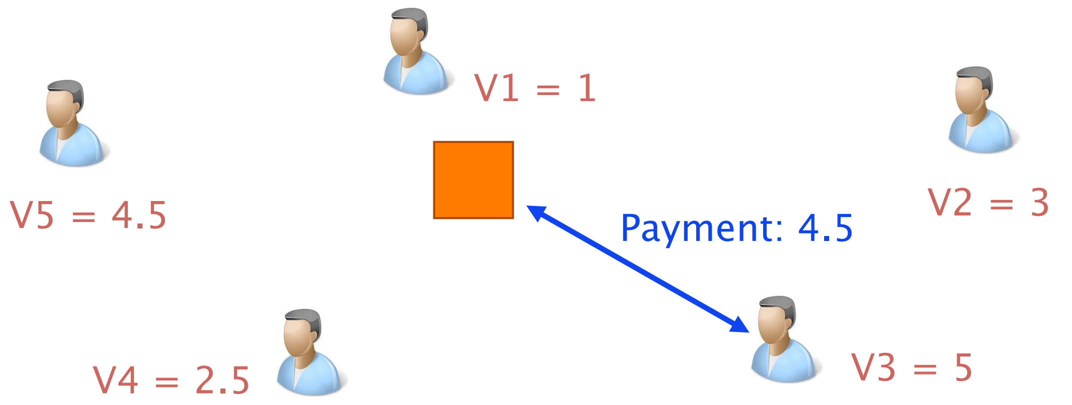
Review

- Online Mechanism Design
  - Why mechanism design?
    - Information Asymmetry



Review

- Online Mechanism Design
  - Objective
    - Elicit the private information of demanders (truthfulness)
    - Achieve desirable (efficient or optimal) allocations based on demanders' truthful information disclosure.
  - Examples (offline mechanism design)
    - Social welfare maximization (efficiency): Second-price Auction
    - Revenue maximization (optimality): Second-price Auction with Optimal Reserve Price

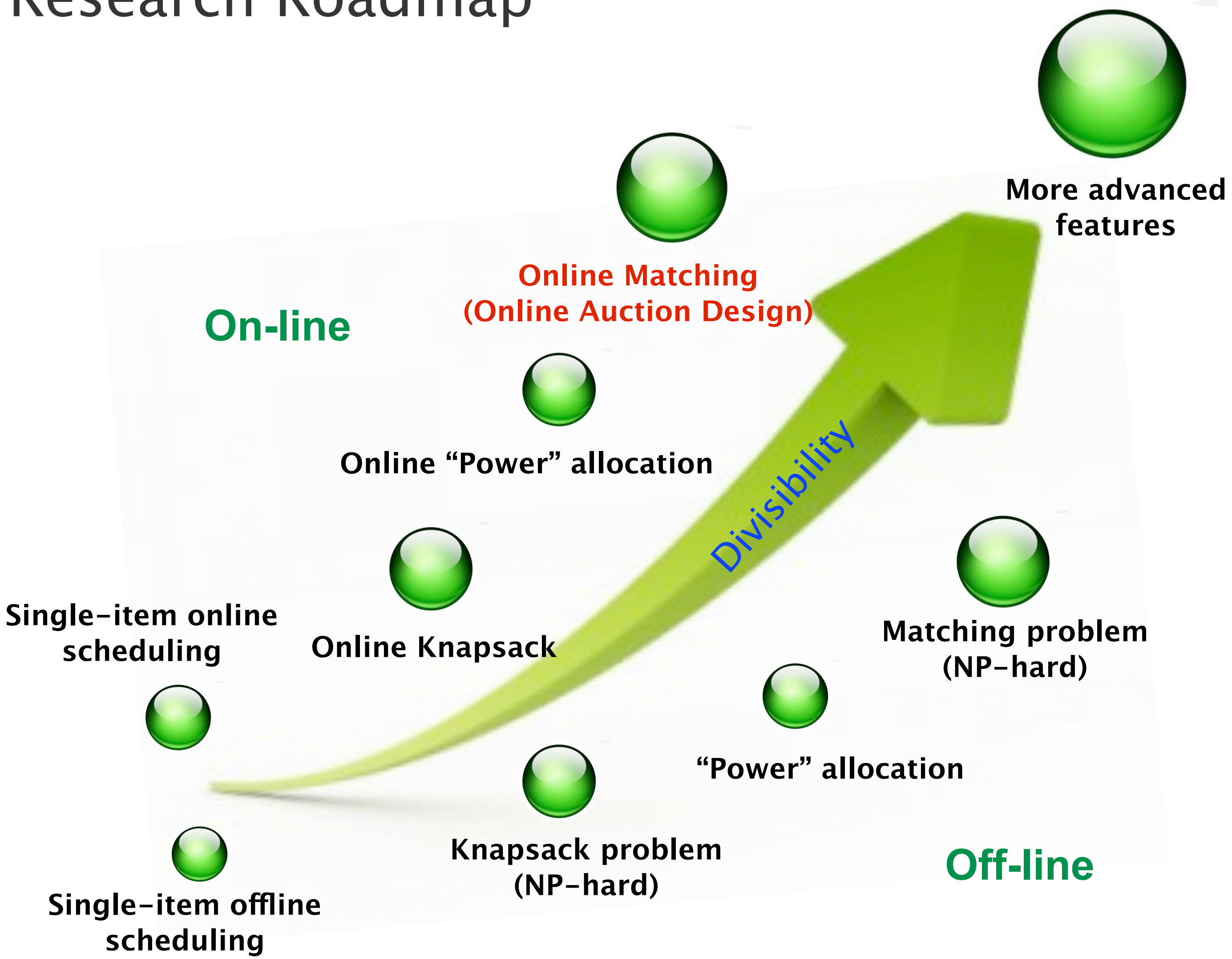


Review

- Online Mechanism Design
  - Objective of Online Mechanism Design
    - Truthfulness
      - Every demander reports truthfully his private information.
    - Constant competitive efficiency
      - The achieved social welfare is constant competitive to the maximum offline welfare.
    - Constant competitive optimality
      - The achieved operator revenue is constant competitive to the maximum offline revenue.

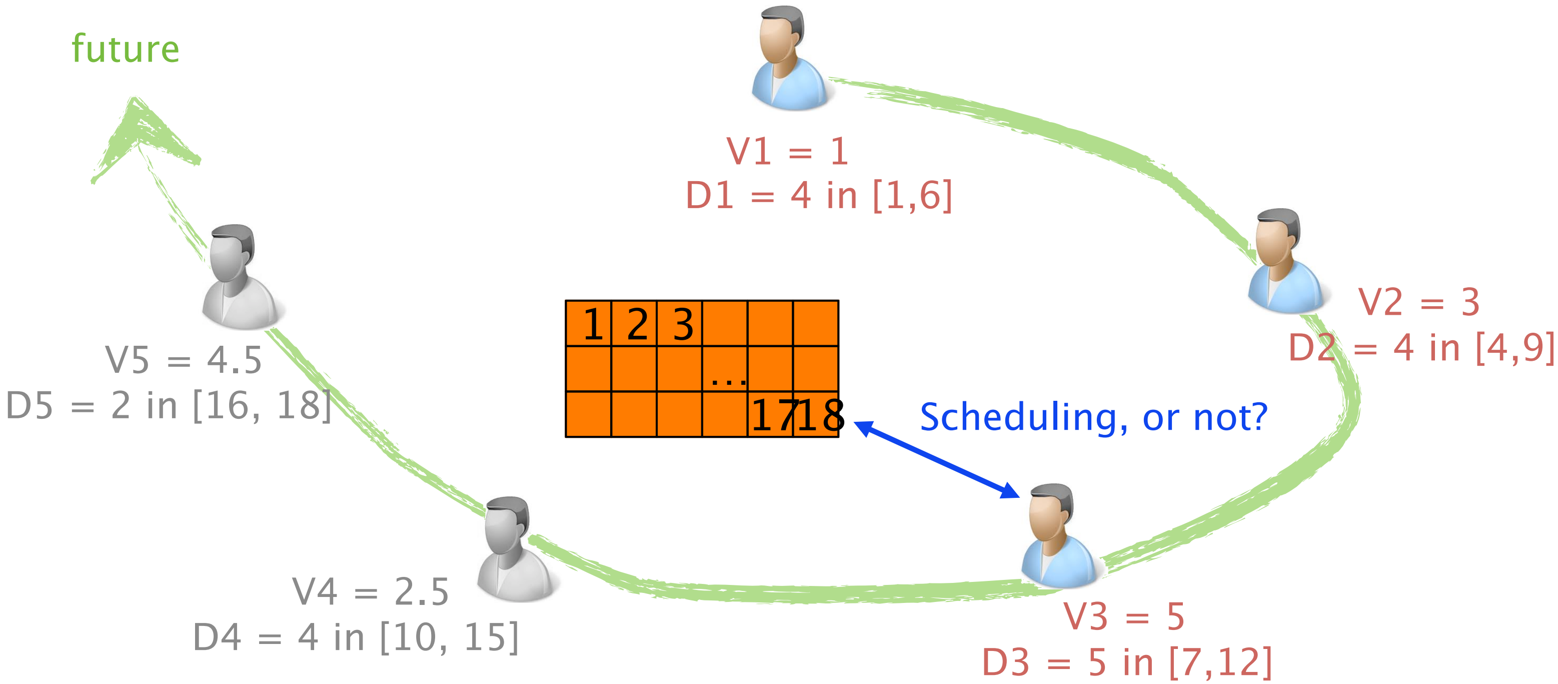
Review

- Research Roadmap



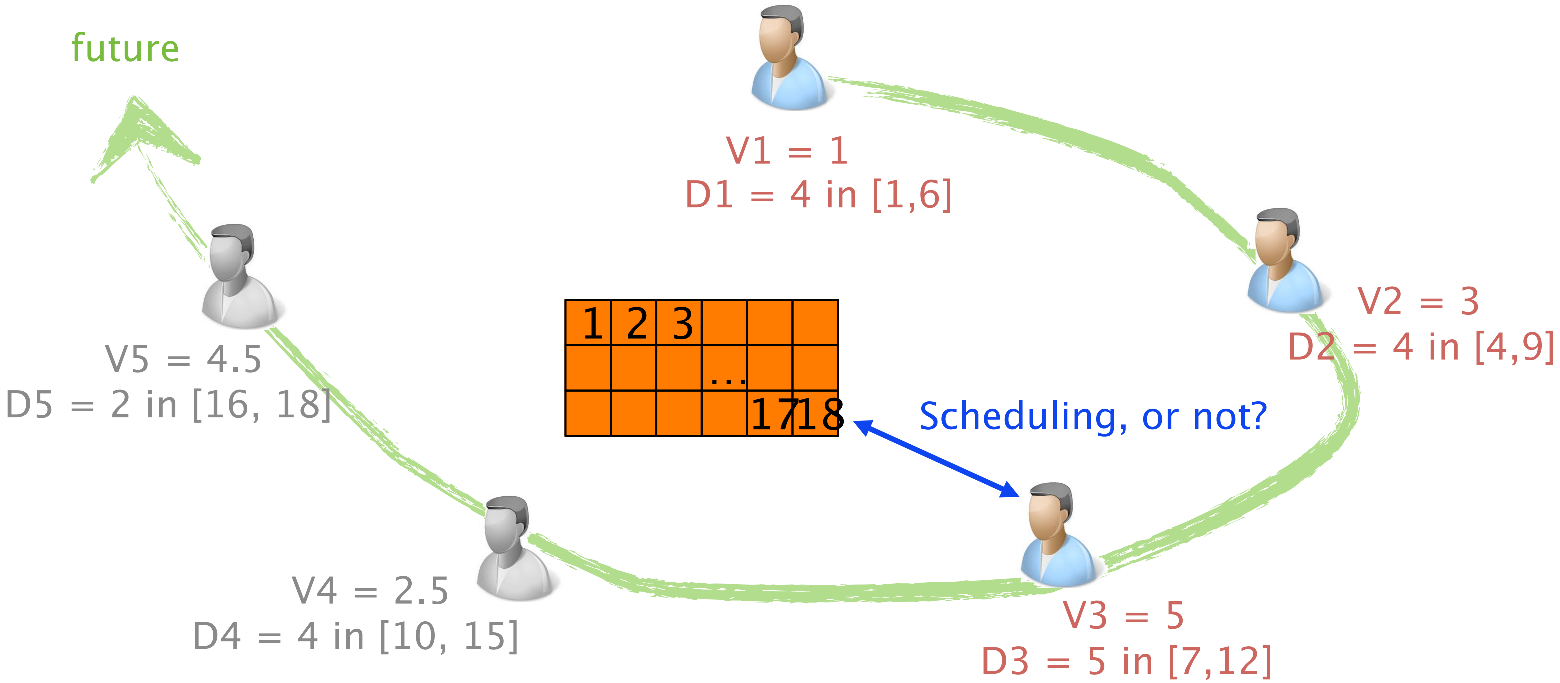
Review

- Online Matching Problem
  - A typical online scheduling problem
  - NP-hard (even for offline matching)



Review

- Online Matching under Information Asymmetry
  - Private information: Vi, Di, [Ai, Bi]
  - Online Auction Mechanism
    - [Mohammad T. Hajiaghayi, 2005 EC]



Outline

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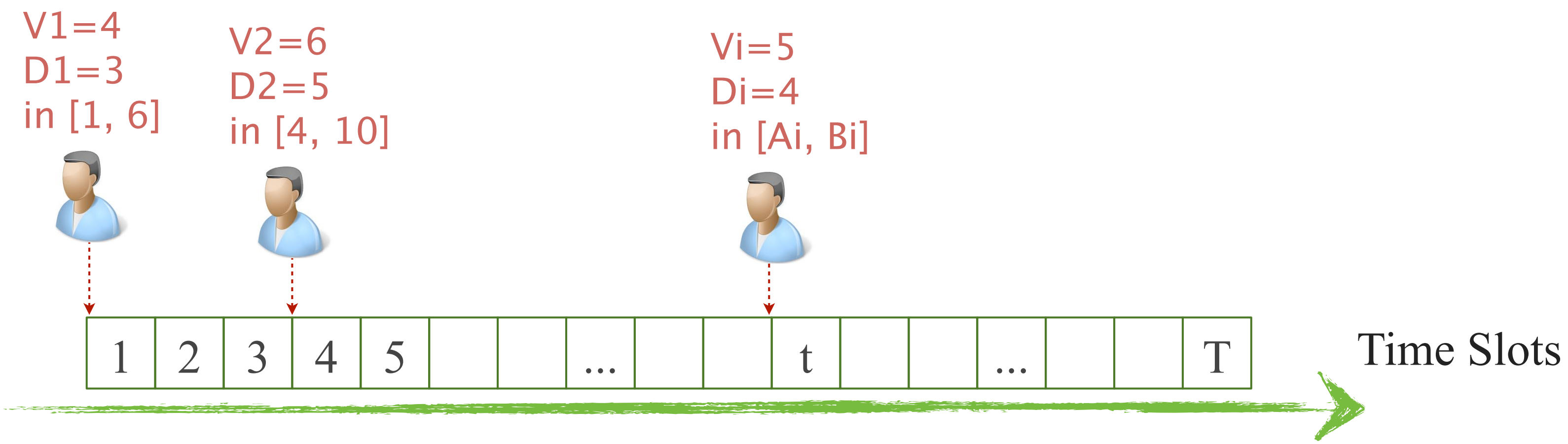
  - System Model
  - Online Auction Mechanism
- Conclusion

System Model

- Mobile Users
  - Arrive randomly and sequentially;
  - For each mobile user i,
    - Vi: valuation, Di: demand for slots,
    - [Ai, Bi]: the range of interested slots
- Network Operator
  - Problem: How to allocate/schedule all slots among mobile users under information asymmetry?



System Model

- Assumptions
  - Successive and constant demand: Di = D (public information)
  - No early arrival: ai ≥ Ai
  - No later departure: bi ≤ Bi

- User Type: $W_i \triangleq [A_i, B_i, V_i]$ (private information)

- User Bid: $w_i \triangleq [a_i, b_i, v_i]$

  Subject to: $[a_i, b_i] \in [A_i, B_i]$

- Key feature – Multi-dimensional private information

Online Auction Design

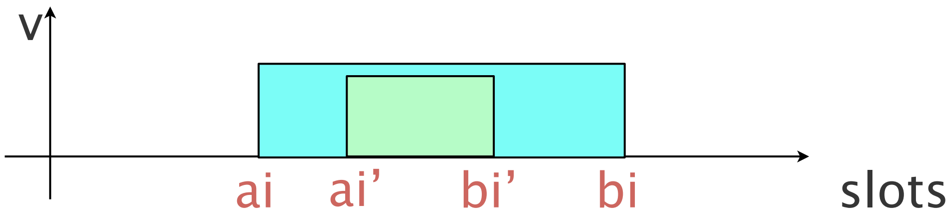
- Online Auction Mechanism Design
  - Design scheduling rule (Q) and payment rule (P)

$$Q: \ [w_1, ..., w_n] \rightarrow [q_1, ..., q_n]$$
$$P: \ [w_1, ..., w_n] \rightarrow [p_1, ..., p_n]$$

  - Truthfulness
    - Every user i reports his truthful type, i.e., wi == Wi.
  - Constant competitive efficiency
    - The achieved social welfare is constant competitive to the maximum offline welfare.
  - Constant competitive optimality
    - The achieved operator revenue is constant competitive to the maximum offline revenue.

Online Auction Design

- Truthfulness
  - Monotonicity: Q is monotone, if qi ≥ qi' for any wi ≥ wi'
    - Here wi ≥ wi' means (i) ai ≤ ai', (ii) bi ≥ bi', and (iii) vi ≥ vi'
    - Generalization of the one-dimensional monotonicity.



  - Monotonicity Criterion: There exists a payment rule P (below) such that the mechanism (Q, P) is truthful, if and only if Q is monotone.

$$p_i(w_i, \boldsymbol{w}_{-i}) = q_i(w_i, \boldsymbol{w}_{-i}) \cdot v_i - \int_0^{v_i} q_i([a_i, b_i, x], \boldsymbol{w}_{-i}) dx \quad (1)$$

The payment rule is value-independent, but not bid-independent !

Online Auction Design

- Truthfulness

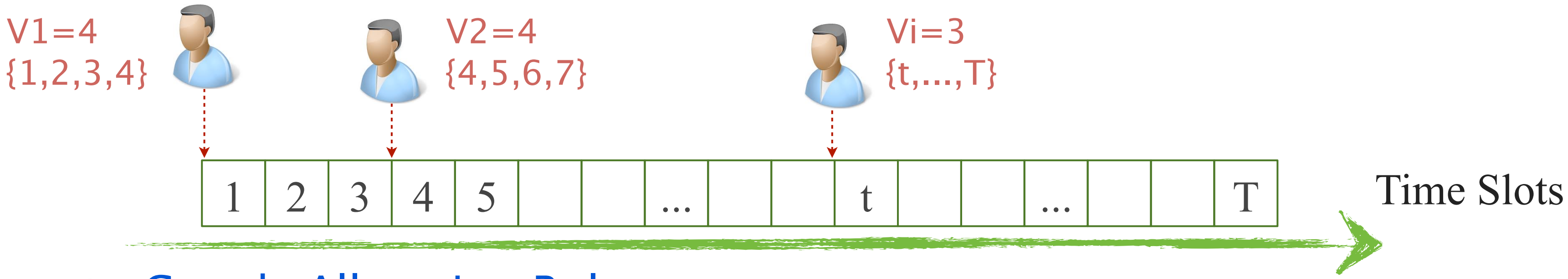
$$p_i(w_i, \boldsymbol{w}_{-i}) = q_i(w_i, \boldsymbol{w}_{-i}) \cdot v_i - \int_0^{v_i} q_i([a_i, b_i, x], \boldsymbol{w}_{-i}) dx \quad (1)$$

User i's payoff

- (i) Suppose ai and bi are fixed: User i will report the truthful vi.
- (ii) Mis-representing a larger ai will decrease user i's payoff;
  Mis-representing a smaller ai is not allowed;
- (iii) Mis-representing a smaller bi will decrease user i's payoff;
  Mis-representing a larger bi is not allowed.

Online Auction Design

- Efficiency
  - Example I – Synchronous Model with Unit Demand D=1



  - Greedy Allocation Rule
    - At the beginning of every time slot: allocate the channel to the bidder with the highest value that is present at that time.
  - Truthful Payment Rule defined in (1)

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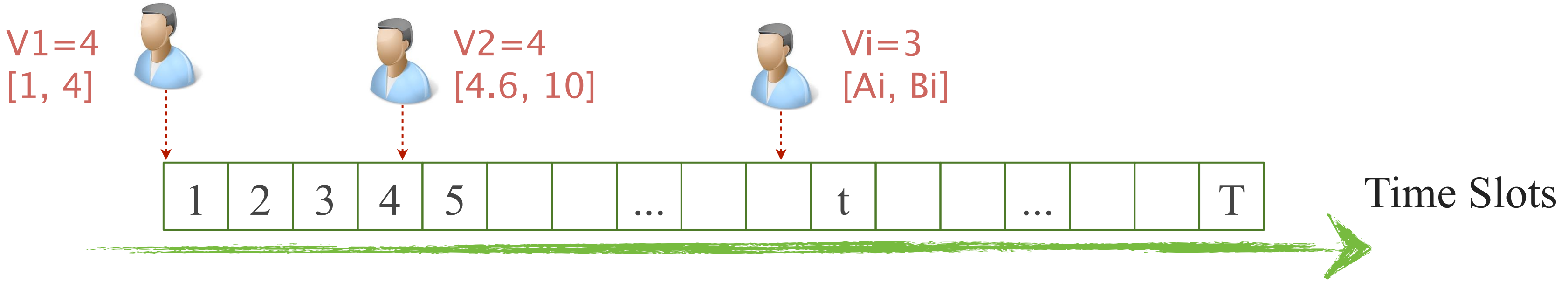
  - Truthfulness
    - Monotonicity Criterion
  - 2-Competitive Efficiency
    - Detailed proof can be referred to online scheduling references.

Online Auction Design

- Efficiency
  - Example II – Asynchronous Model with Unit Demand D=1



  - Modified Greedy Allocation Rule – Preempt and Restart
    - Whenever a new user arrives or an existing user completes its transmission (critical time), reallocate the channel to the bidder with the highest virtual value that is present at that time.
      - Virtual value of bidder i: $\widetilde{v}_i = v_i \cdot 2^{\sigma_i}$
  - Truthful Payment Rule defined in (1)

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  - Truthfulness
  - 5-Competitive Efficiency

Online Auction Design

- Optimality
  - There is no truthful auction mechanism whose revenue is constant-competitive.
  - Relaxation of Information Asymmetry
    - For example, suppose $w_i \in [\underline{w}_i, \bar{w}_i]$, and the operator know the upper bound. There is an online auction mechanism which achieves a competitive ratio of

$$O(\log(h)) \quad \text{where} \quad h = \frac{\bar{w}_i}{\underline{w}_i}$$

Conclusion

- We review the basic concepts of online scheduling and online mechanism design problem.
- We present an online auction mechanism design (by Mohammad) for online matching problems:
  - Truthfulness
  - Efficiency
  - Optimality
- Future extensions
  - Different and non-unit user demands
  - Non-successive user demand
  - Multiple channels
  - ...

Most of these extensions are open problem.