Population Game

— Theory, Examples, and Applications

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Outline

- Population Game Theory
- Real World Examples
- Our Applications
  - TV White Space Information Market
  - User-Provided Network
  - Wi-Fi Community Network
  - Peer-to-Peer Mobile Crowd Sensing

Theory

Evolutionary Game Theory

- Evolutionary Game Theory considers a population decision makers (players), wherein the frequency with which a particular decision is made can be time varying. It is a theory started from Biology.

- Some interesting and important questions:
  - What is the stable population (equilibrium)?
  - Will the population evolve toward the equilibrium?
  - Will the population move away from the equilibrium (stability of equilibrium)?

Population Game

- Elements in Population Game
  - Player: An infinite population of individuals;
  - Strategy: Each player can use a set of pure strategies *S*, or a mixed strategy σ over *S*;
  - Payoff: Each player's payoff depends on population profile *X*.

Population Game

- Population Profile — $\boldsymbol{x}$

Definition

Consider an infinite population of individuals that can use a set of pure strategies, $\boldsymbol{S}$. A population profile is a vector $\boldsymbol{x}$ that gives a probability $x(s)$ with which each strategy $s \in \boldsymbol{S}$ is played in the population.

- Example
  - Suppose that a population can choose $S = \{s_1, s_2\}$
  - Case 1: Half population chooses s_1, and the other half chooses s_2, then $X = (0.5, 0.5)$;
  - Case 2: All population chooses the mixed strategy $(0.5, 0.5)$, then $X = (0.5, 0.5)$;

Population Game

- **Payoff** of Individual

Definition

Consider a particular individual in the population with profile $\boldsymbol{x}$. If that individual uses a strategy σ, the individual's payoff is denoted as $\pi(\sigma, \boldsymbol{x})$. The payoff of this strategy is

$$\pi(\sigma, \boldsymbol{x}) = \sum_{s \in \boldsymbol{S}} p(s) \pi(s, \boldsymbol{x}).$$

Population Game

- Type 1: Games against the field

Definition

A game against the field is one in which there is no specific “opponent” for a given individual - their payoff depends on what everyone in the population is doing.

- Type 2: Games with pairwise contests

Definition

A pairwise contest game describes a situation in which a given individual plays against an opponent that has been randomly selected (by nature) from the population and the payoff depends just on what both individual do.

Equilibrium

- An equilibrium of a population game is the end points of the evolution of the population (also called stable population, evolutionary stability).

Theorem

Let σ^* be a strategy that generates a population profile $\boldsymbol{x}^*$. Let $\boldsymbol{S}^*$ be the support for σ^*. If the population is stable, then

$$\sigma^* \in \mathit{argmax}_{\sigma \in \sum} \{\pi(\sigma, \boldsymbol{x}^*)\}.$$

$$\text{and} \quad \pi(s, \boldsymbol{x}^*) = \pi(\sigma^*, \boldsymbol{x}^*), \forall s \in \boldsymbol{S}^*.$$

- At the equilibrium, the strategy adopted by each individual must be the best response to the population profile.

Stability of Equilibrium

- **Stability of Equilibrium**
  - The robustness of an equilibrium to a small change on population profile.

Comment on the above theorem

If σ^* is unique best response to $\boldsymbol{x}^*$, then the evolution of the population clearly stops. But if it is not unique, so there are some other strategies that do equally well in the population with profile $\boldsymbol{x}^*$, then the population could drift in the direction of other strategy and its corresponding population profile. We want to understand this situation.

Stability of Equilibrium

- **Post-Entry Population**

Definition

Consider a population where initially all the individuals adopt some strategy σ^*. Suppose a mutation occurs and a small proportion ϵ of individuals use some other strategy σ. The new population is called the post-entry population and will be denoted as $\mathbf{x}_\epsilon$.

- **Example**
  - Consider a population with $S = \{s_1, s_2\}$ and $X^* = (0.5, 0.5)$
  - Suppose the mutant strategy is (0.75, 0.25), then

$$\mathbf{x}_\epsilon = (1-\epsilon)\sigma^* + \epsilon\sigma = (1-\epsilon)\left(\frac{1}{2}, \frac{1}{2}\right) + \epsilon\left(\frac{3}{4}, \frac{1}{4}\right) = \left(\frac{1}{2} + \frac{\epsilon}{4}, \frac{1}{2} - \frac{\epsilon}{4}\right)$$

Stability of Equilibrium

- **Evolutionary Stable Strategy (ESS)**
  - Also called **stable equilibrium.**

Stability of ESS

A mixed strategy σ^* is an evolutionary stable strategy (ESS) if there exists an $\bar{\epsilon}$ such that for every $0 < \epsilon < \bar{\epsilon}$ and every $\sigma \neq \sigma^*$

$$\pi(\sigma^*, \boldsymbol{x}_\epsilon) > \pi(\sigma, \boldsymbol{x}_\epsilon).$$

- **Physical meaning**: A strategy is an ESS if mutants that adopt any other strategy **achieve a worst payoff** in the post-entry population, provided that the proportion of mutants is sufficiently small.

Examples

Example A

- **Real World Example — Population Ratio**

Example of a Game Against the field

Why the population ratio (male and female) is 50 : 50?

- The proportion of males (females) in the population is μ $(1 - \mu)$.
- Each female mates once and produce n children.
- Males mates, on average, $(1 - \mu)/\mu$ times.
- Only female genes affect the sex ratio of offspring.
- Assume females available strategies are (a) s_1: produce male offspring; (b) s_2: produce female offspring. The general strategy $\sigma = (p, 1 - p)$ produces a proportion p of male offspring.
- The current population profile is $\boldsymbol{x} = (\mu, 1 - \mu)$
- What is the Evolutionary Stable Strategy (ESS)?

Example A

- **Real World Example — Population Ratio**

Solution

- Since payoff of children is n, let's consider the number of grandchildren. Given the population profile $\boldsymbol{x} = (\mu, 1-\mu)$, the payoffs are

$$\pi(s_1, \boldsymbol{x}) = n^2 \left(\frac{1-\mu}{\mu} \right) \quad ; \quad \pi(s_2, \boldsymbol{x}) = n^2.$$

- The expected payoff for the strategy σ is

$$\pi(\sigma, \boldsymbol{x}) = n^2 \left(\frac{1-\mu}{\mu} \right) p + n^2(1-p).$$

  Because n is independent of the strategy chosen, we can set $n = 1$ (since we are only interested in the population ratio).

Example A

- **Real World Example — Population Ratio**

Solution: continue

To find an ESS, consider the following cases:

- If $\mu < 1/2$, then using s_1 will have more grandchildren which eventually cause μ to increase. So s_1 is not an ESS.
- If $\mu > 1/2$, then using s_2 have more grandchildren causing μ to fall. So s_2 is not an ESS.
- Is $\sigma^* = (\frac{1}{2}, \frac{1}{2})$ a potential ESS? Let use the ESS Stability Theorem, which states that

$$\pi(s_1, \boldsymbol{x}^*) = \pi(s_2, \boldsymbol{x}^*) = \pi(\sigma^*, \boldsymbol{x}^*).$$

  So if the population profile is $\boldsymbol{x}^* = (\frac{1}{2}, \frac{1}{2})$, then $\sigma^* = (\frac{1}{2}, \frac{1}{2})$ is an ESS (note that this is only a necessary condition).

Example A

- **Real World Example — Population Ratio**

Solution: continue

Let us show the "sufficient" condition that $\sigma^* = (\frac{1}{2}, \frac{1}{2})$ is indeed an ESS.

- Let $\sigma = (p, 1-p)$ be another strategy, then

$$\boldsymbol{x}_\epsilon = (1-\epsilon)\sigma^* + \epsilon\sigma$$

$$\text{so,} \quad \mu_e = (1-\epsilon)\frac{1}{2} + \epsilon p = \frac{1}{2} + \epsilon\left(p - \frac{1}{2}\right).$$

- The ESS condition is $\pi(\sigma^*, \boldsymbol{x}_\epsilon) > \pi(\sigma, \boldsymbol{x}_\epsilon)$ where

$$\begin{aligned} \pi(\sigma^*, \boldsymbol{x}_\epsilon) &= \frac{1}{2} + \frac{1}{2}\left(\frac{1-\mu_\epsilon}{\mu_\epsilon}\right) \\ \pi(\sigma, \boldsymbol{x}_\epsilon) &= (1-p) + p\left(\frac{1-\mu_\epsilon}{\mu_\epsilon}\right). \end{aligned}$$

Example A

- **Real World Example — Population Ratio**

Solution: continue

- The difference

$$\begin{aligned}\pi(\sigma^*, \boldsymbol{x}_\epsilon) - \pi(\sigma, \boldsymbol{x}_\epsilon) &= (p - \frac{1}{2}) + (\frac{1}{2} - p)\left(\frac{1-\mu_\epsilon}{\mu_\epsilon}\right) \\ &= (\frac{1}{2} - p)\left[\frac{1-\mu_\epsilon}{\mu_\epsilon} - 1\right] \\ &= (\frac{1}{2} - p)\left[\frac{1-2\mu_\epsilon}{\mu_\epsilon}\right].\end{aligned}$$

If the difference is positive for $\sigma = (p, 1-p)$, with $p \neq 1/2$, then σ^* is an ESS.

Example A

- Real World Example — Population Ratio

In the population ratio game, (0.5, 0.5) is the unique ESS.

Example B

- **Real World Example — Social Network**

Example

Consider a "simplified" Internet. There are two operating systems available: L and W. A user of window W has a basic utility of 1, but L is a better operating system so a user of L has a basic utility of 2. If two computers have the same operating system, then they can communicate over the network. A user's utility rises linearly with the proportion of computers that can be communicated with, up to a maximum increment of 2. Let x be the proportion of $W-$users, then $\pi(W, x) = 1 + 2x$ and $\pi(L, x) = 2 + 2(1 - x)$. What are the ESSs in this population game?

Example B

- **Real World Example — Social Network**

Solution

- Potential ESSs are:
  - σ_W: everyone uses W, then $x = 1$ and $\pi(W, 1) > \pi(L, 1)$.
  - σ_L: everyone uses L, then $x = 0$ and $\pi(L, 0) > \pi(W, 0)$.
  - σ_m: mixed strategy in which W is used $3/4$ of the time, then $x = 3/4$ and $\pi(W, 3/4) = \pi(L, 3/4)$.
- Now $\boldsymbol{x}_\epsilon = (p^* + \epsilon(p - p^*), 1 - p^* - \epsilon(p - p^*))$. So

$$
\begin{aligned}
\delta_\pi &= \pi(\sigma^*, \boldsymbol{x}_\epsilon) - \pi(\sigma, \boldsymbol{x}_\epsilon) \\
&= p^*\pi(W, \boldsymbol{x}_\epsilon) + (1 - p^*)\pi(L, \boldsymbol{x}_\epsilon) - p\pi(W, \boldsymbol{x}_\epsilon) - (1 - p)\pi(L, \boldsymbol{x}_\epsilon) \\
&= (p^* - p)\left(\pi(W, \boldsymbol{x}_\epsilon) - \pi(L, \boldsymbol{x}_\epsilon)\right) \\
&= (p^* - p)(4p^* - 3 - 4\epsilon(p^* - p))
\end{aligned}
$$

Example B

- **Real World Example — Social Network**

Solution: continue

Taking each candidate ESSs in turn:

- σ_W: $p^* = 1$, so

$$\delta_\pi = (1-p)(1-4\epsilon(1-p)) > 0, \ \forall p \neq 1, \ \text{and } \epsilon < \bar{\epsilon} = 1/4.$$

So it is an ESS.

Example B

- **Real World Example — Social Network**

Solution: continue

Taking each candidate ESSs in turn:

- σ_L: $p^* = 0$, so

$$\delta\pi = p(3 - 4\epsilon p) > 0, \ \forall p \neq 0, \ \text{and } \epsilon < \bar{\epsilon} = 3/4.$$

So σ_L is an ESS.

Example B

- **Real World Example — Social Network**

Solution: continue

Taking each candidate ESSs in turn:

- σ_m: $p^* = 3/4$, so

$$\delta_\pi = -4\epsilon\left(\frac{3}{4} - p\right)^2 < 0, \forall p \neq \frac{3}{4} \text{ and } \epsilon > 0.$$

  So it is “**not**” an ESS.

Example B

- Real World Example — Social Network

In the social network game, there are three equilibrium points: (1, 0), (0, 1), and (3/4, 1/4). The first two equilibria are ESS.

Example C

- **Real World Example — Currency War**

Example: The evolution of money

- In an remote island, inhabitants have to decide to use either "beads" or "shells" as tokens of money in commerce.
- A transaction is only successful if both parties use the same form of token.
- Assume that a trader gets a utility increment of 1 if the transaction is successful and 0 if it fails.
- The general strategy to an individual is to use beads with p, i.e., $\sigma = (p, 1 - p)$. The population profile $\boldsymbol{x} = (x, 1 - x)$.
- What is an ESS ?

Example C

- **Real World Example — Currency War**

Solution

- An individual attempts to trade with a randomly selected member of the population, his payoff

$$\pi(\sigma, \boldsymbol{x}) = px + (1-p)(1-x) = (1-x) + p(2x-1).$$

We see that

$$x > \frac{1}{2} \longrightarrow \hat{p} = 1 \quad \text{and} \quad p = 1 \longrightarrow x = 1.$$

So $\sigma_b^* = (1, 0)$ is a potential ESS with $\boldsymbol{x} = (1, 0)$.

- The post-entry population is:

$$\boldsymbol{x}_\epsilon = (1-\epsilon)(1,0)+\epsilon(p, 1-p) = (1-\epsilon(1-p), \epsilon(1-p)).$$

Example C

- **Real World Example — Currency War**

Solution: continue

- In this population, the payoff for an arbitrary strategy is

$$\pi(\sigma, \boldsymbol{x}_\epsilon) = \epsilon(1-p) + p(1 - 2\epsilon(1-p)).$$

- The payoff for the candidate ESS is $\pi(\sigma_b^*, \boldsymbol{x}_\epsilon) = 1 - \epsilon(1-p)$, so

$$\begin{aligned} \pi(\sigma_b^*, \boldsymbol{x}_\epsilon) - \pi(\sigma, \boldsymbol{x}_\epsilon) &> 0, \\ \Longleftrightarrow (1-p)(1-2\epsilon(1-p)) &> 0. \end{aligned}$$

- Now, $\forall p \neq p^*$, we have $(1-p) > 0$, so σ_b^* is an ESS if and only iff $\epsilon(1-p) < \frac{1}{2}$. That is $\bar{\epsilon} = \frac{1}{2}$.

Example C

- **Real World Example — Currency War**

Solution: continue

- The strategy $\sigma_s^* = (0, 1)$ is another ESS because the post-entry population,

$$\boldsymbol{x}_\epsilon = (\epsilon p, 1 - \epsilon p),$$

the payoff for an arbitrary strategy is

$$\pi(\sigma, \boldsymbol{x}_\epsilon) = (1 - \epsilon p) - p(1 - 2\epsilon p),$$

and the payoff for the candidate ESS is

$$\pi(\sigma_b^*, \boldsymbol{x}_\epsilon) = 1 - \epsilon p.$$

- We have:

$$\pi(\sigma_b^*, \boldsymbol{x}_\epsilon) - \pi(\sigma, \boldsymbol{x}_\epsilon) > 0 \iff p(1 - 2\epsilon p) > 0.$$

- Now, $\forall p \neq p^*$, we have $p > 0$, so σ_s^* is an ESS if and only if $\epsilon p < \frac{1}{2}$, i.e., $\bar{\epsilon} = \frac{1}{2}$.

Example C

- **Real World Example — Currency War**

Solution: continue

The final candidate for an ESS is $\sigma_m^* = (\frac{1}{2}, \frac{1}{2})$ because

$$x = \frac{1}{2} \Longrightarrow \hat{p} \in [0, 1] \Longrightarrow x \in [0, 1].$$

(including, of course, $x = 1/2$). Consider the post-entry population

$$\boldsymbol{x}_\epsilon \;=\; (1-\epsilon)\left(\frac{1}{2}, \frac{1}{2}\right) + \epsilon(p, 1-p) = \left(\frac{1}{2} - \frac{1}{2}\epsilon(1-2p), \frac{1}{2} + \frac{1}{2}\epsilon(1-2p)\right).$$

The payoff for an arbitrary strategy is $\pi(\sigma, \boldsymbol{x}_\epsilon) = \frac{1}{2} + \frac{1}{2}\epsilon(1-2p)^2$, and the payoff for the candidate ESS is $\pi(\sigma_m^*, \boldsymbol{x}_\epsilon) = \frac{1}{2}$. So

$$\pi(\sigma_m^*, \boldsymbol{x}_\epsilon) - \pi(\sigma, \boldsymbol{x}_\epsilon) > 0 \Longleftrightarrow -\frac{1}{2}\epsilon(1-2p)^2 > 0.$$

Because $\epsilon > 0$ and $p \neq \frac{1}{2}$, this condition **cannot be satisfied**; so σ_m^* is **not** an ESS. So whether to use beads or shells depends on the initial condition.

Example C

- Real World Example — Currency War

In the currency war game, there are three equilibrium points: (1, 0), (0, 1), and (1/2, 1/2). The first two equilibria are ESS.

Example D

- **Example of Game with Pairwise Contests**

The Hawk-Dove Game

- Individuals can use one of two possible pure strategies: (a) H: be aggressive, (b) D: be non-aggressive.
- In general, individual can use a randomized strategy $\sigma = (p, 1-p)$ with probability p of using H.
- A population consists of individuals that are aggressive with probability x, i.e., $\mathbf{x} = (x, 1-x)$, this can arise because
  - a monomorphic population, everyone uses $\sigma = (x, 1-x)$, or
  - a polymorphic population, a fraction x of population use $\sigma_H = (1, 0)$ and a fraction $1-x$ use $\sigma_D = (0, 1)$. Let consider only monomorphic population.

Example D

- **Example — The Hawk-Dove Game**

The Hawk-Dove Game: continue

- There is a resource (e.g., food, breeding site,...etc) with value v. The outcome of a conflict depends on the types of two individuals that meet.
- Possible combinations:
  1. a hawk and a dove: hawk wins,
  2. a dove and a dove: they "share" the resource evenly,
  3. a hawk and a hawk: they fight with one winner gets the resource and the other loser pays a cost (i.e., injury) of c.
- What is the outcome of the game? What is the ESS?

Example D

- **Example — The Hawk-Dove Game**

Solution

- The payoff of an individual:

$$\pi(\sigma, \boldsymbol{x}) = px\frac{v-c}{2} + p(1-x)v + (1-p)(1-x)\frac{v}{2}.$$

Example D

- **Example — The Hawk-Dove Game**

Solution

- Assume $v < c$, there is no pure-strategy ESS. Why?
  - In a population of Doves ($x = 0$),

$$\pi(\sigma, \boldsymbol{x}_D) = pv + (1-p)\frac{v}{2} = (1+p)\frac{v}{2}.$$

    It is best to set $p = 1$ (play hawk). As a consequence, the proportion of more aggressive individual will increase.
  - In a population of Hawks ($x = 1$),

$$\pi(\sigma, \boldsymbol{x}_H) = p\frac{v-c}{2}.$$

    It is best to set $p = 0$ because $(v - c) < 0$. As a consequence, the proportion of less aggressive individual will increase.

Example D

- **Example — The Hawk-Dove Game**

Solution: continue

- Is there a mixed strategy ESS, $\sigma^* = (p^*, 1 - p^*)$? For σ^* to be ESS, it must be a best response to the population $\boldsymbol{x}^* = (p^*, 1 - p^*)$ that it generates.
- If $p^* = v/c$, then any choice of p (including p^*) gives the same payoff, so we have

$$\sigma^* = \left(\frac{v}{c}, 1 - \frac{v}{c}\right),$$

as a candidate ESS when $v < c$.

Example D

- **Example — The Hawk-Dove Game**

- To confirm σ^* is an ESS, we must show that for $\sigma = (p, 1-p) \neq \sigma^*$, $\pi(\sigma^*, \boldsymbol{x}_\epsilon) > \pi(\sigma, \boldsymbol{x}_\epsilon)$, where

$$\begin{aligned} \boldsymbol{x}_\epsilon &= ((1-\epsilon)p^* + \epsilon p, ((1-\epsilon)(1-p^*) + \epsilon(1-p)) \\ &= (p^* + \epsilon(p - p^*), 1 - p^* + \epsilon(p^* - p)). \end{aligned}$$

- We have

$$\begin{aligned} \pi(\sigma^*, \boldsymbol{x}_\epsilon) &= p^*(p^* + \epsilon(p - p^*))\frac{v-c}{2} + p^*(1 - p^* + \epsilon(p^* - p))v + \\ &\quad (1-p^*)(1 - p^* + \epsilon(p^* - p))\frac{v}{2}, \\ \pi(\sigma, \boldsymbol{x}_\epsilon) &= p(p^* + \epsilon(p - p^*))\frac{v-c}{2} + p(1 - p^* + \epsilon(p^* - p))v + \\ &\quad (1-p)(1 - p^* + \epsilon(p^* - p))\frac{v}{2}. \end{aligned}$$

- Substituting $p^* = v/c$, we have

$$\pi(\sigma^*, \boldsymbol{x}_\epsilon) - \pi(\sigma, \boldsymbol{x}_\epsilon) = \frac{\epsilon c}{2}(p^* - p)^2 > 0.$$

so $\sigma^* = (p^*, 1-p^*)$ is an ESS.

Example D

- Example — The Hawk-Dove Game

In the hawk-dove game with V<C, there is a unique ESS: (V/C, 1-V/C).

Our Applications

Model Extension

- In the basic population game model, all of the game players are homogeneous (i.e., with the same strategy set and payoff function).

- In a population game with homogeneous players, we can focus on the symmetric equilibrium, wherein all players choose the same strategy (hence equals the population profile X).
  - For any asymmetric equilibrium, we can always find an equivalent symmetric equilibrium.

Model Extension

- In practice, a population game may consist of a population of heterogeneous players.
- Example: Modified Social Network
  - Two social networks: L and W
  - The whole population is divided into two types: Q_1 and Q_2, depending on their evaluations for the population profile:
    - For type Q_1: Payoff = $1 + Q_1 * x$, or $2 + Q_1 * (1 - x)$
    - For type Q_2: Payoff = $1 + Q_2 * x$, or $2 + Q_2 * (1 - x)$
  - Suppose the population profile of type-Q_1 players is $(x_1, 1-x_1)$, and the population profile of type-Q_2 players is $(x_2, 1-x_2)$;
  - Then, the entire population profile $(x, 1-x)$ is given by:
  - $$x = n_1 * x_1 + n_2 * x_2$$

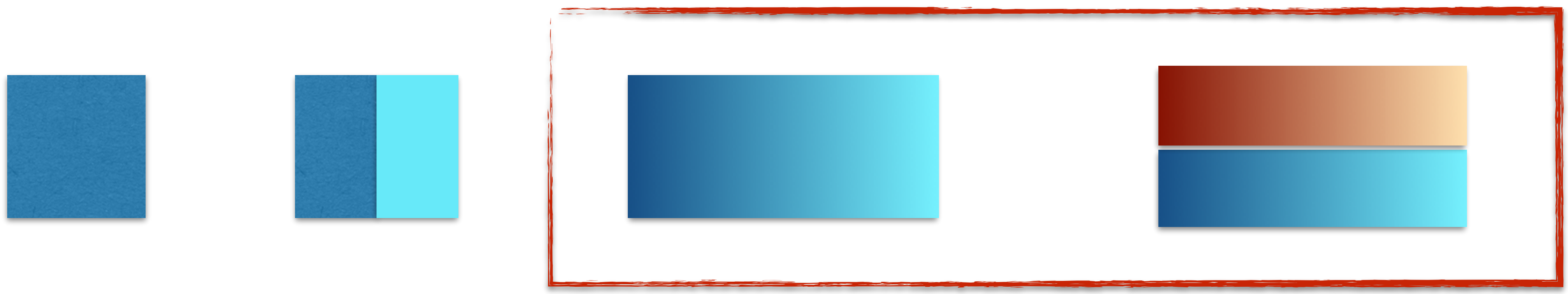
Model Extension

- Population Game Model Extensions
  - Homogeneous Players (One Type)
  - Heterogeneous Players with Finite Types
  - Heterogeneous Players with Infinite Types (Single-Dimension)
  - Heterogeneous Players with Infinite Types (Multi-Dimension)

Model Extension

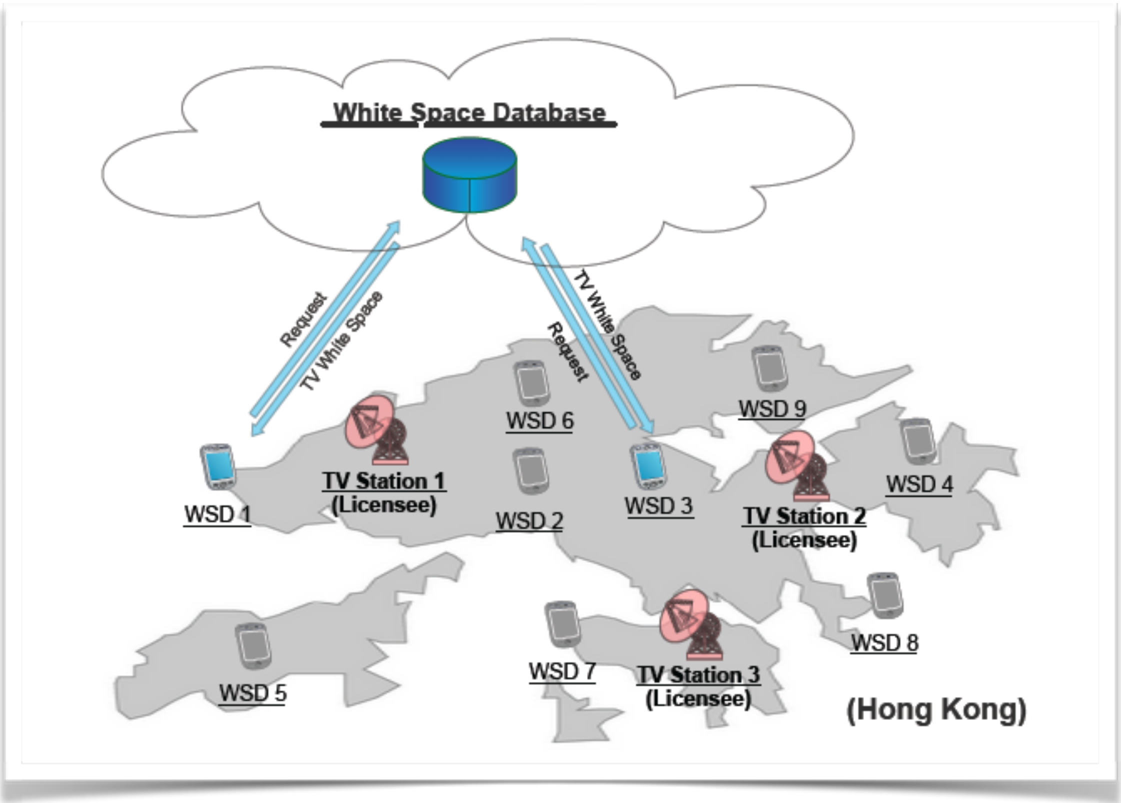
- Population Game Model
  - Homogeneous Players (One Type)
  - Heterogeneous Players with Finite Types
  - Heterogeneous Players with Infinite Types (Single-Dimension)
  - Heterogeneous Players with Infinite Types (Multi-Dimension)

- Our Focus



Application 1

- TV White Space Information Market
  - An infinite population of heterogenous WSDs want to access Internet through TV channels;
  - Choose to purchase information from database or not.



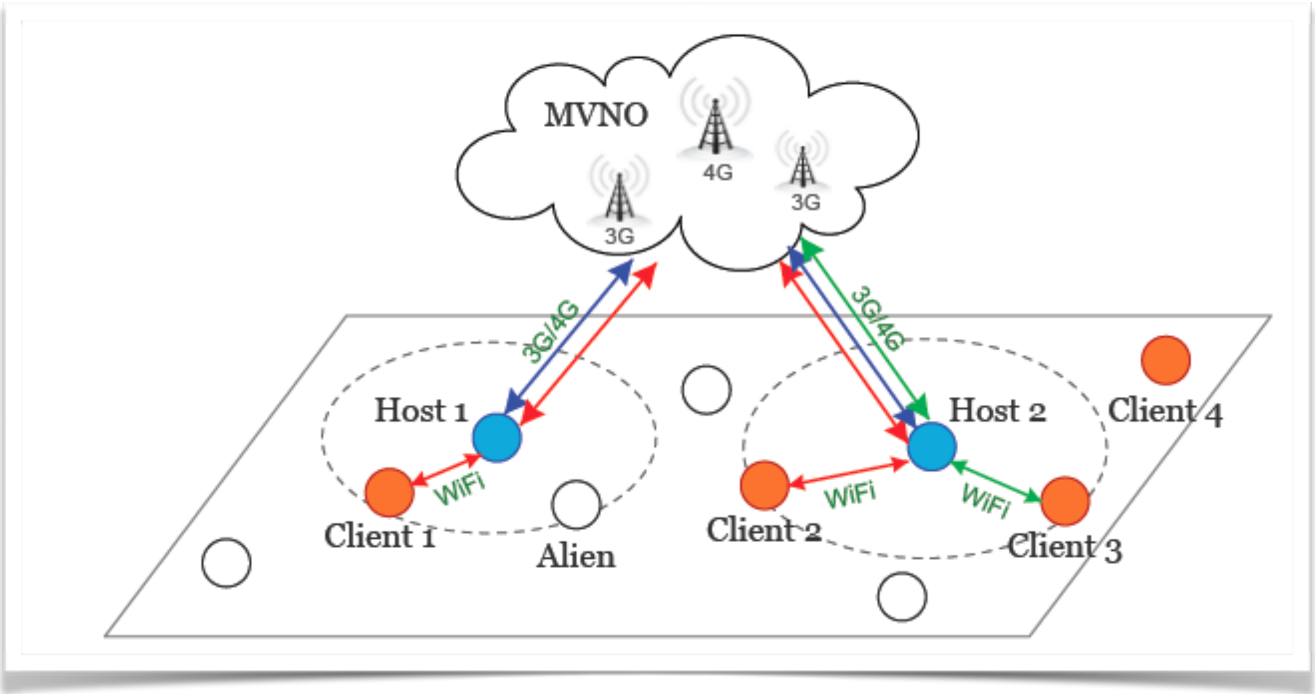
Application 1

- TV White Space Information Market
  - WSDs are heterogenous, with different information evaluations Q in [Q_{low}, Q_{up}]; (infinite and one-dimensional type)
  - Population profile: (x, 1-x)
    - A population x of WSDs choose to purchase, and the remaining population 1-x choose not to purchase.
  - Payoff of each WSD:
    - (1) Information valuation minus price, if purchasing information,
    - (2) zero, if not purchasing the information.

  - Key Problem: Equilibrium Analysis
    - What is the equilibrium population?
    - Is the equilibrium stable?

Application 2

- User-Provided Network
  - An infinite population of heterogenous mobile users want to access Internet through MVNO network;
  - Choose to become hosts, clients, or aliens.

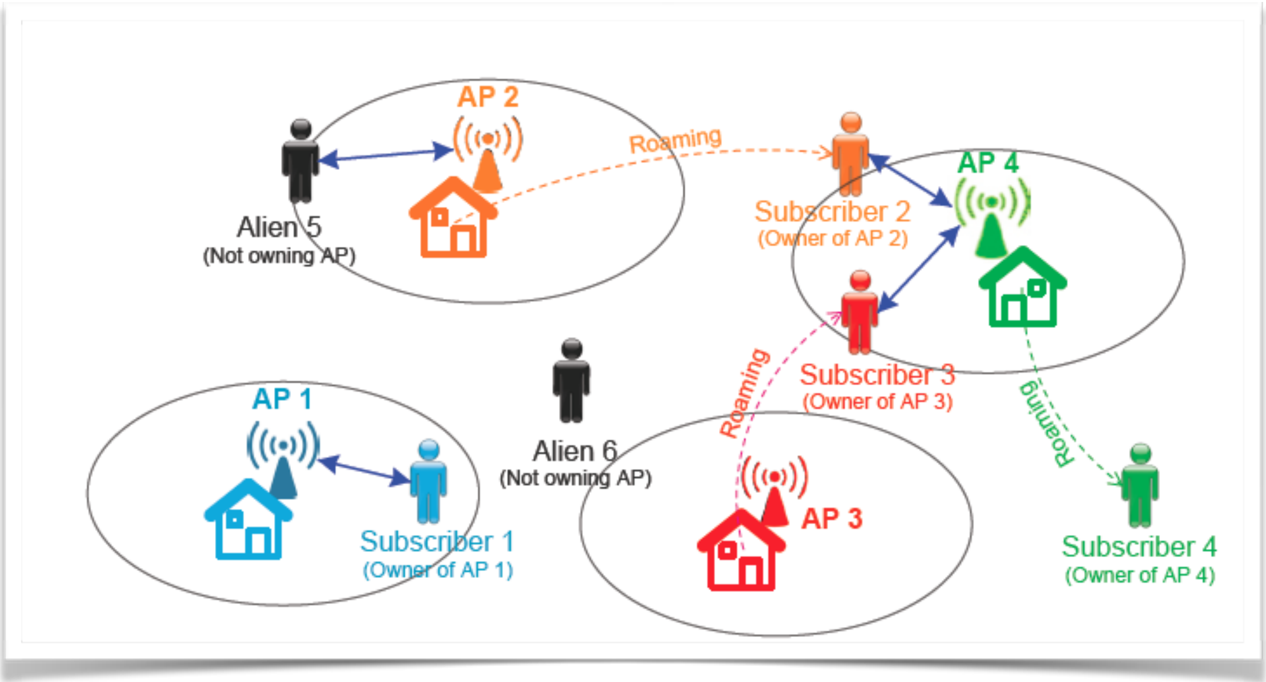


Application 2

- User-Provided Network
  - Users are heterogenous, with different service request probability Q in [0, 1]; (infinite and one-dimensional type)
  - Population profile: $(x_h, x_c, 1-x_h-x_c)$
    - A population x_h (x_c) of users choose to be hosts (clients), and the remaining population $1-x_h-x_c$ choose to be aliens.
  - Payoff of each WSD:
    - (1) Payoff of a host increases with x_c, and decreases with x_h ;
    - (2) Payoff of a client increases with x_h, and decreases with x_c ;
  - Key Problem: Equilibrium Analysis
    - What is the equilibrium population?
    - Is the equilibrium stable?

Application 3

- **Wi-Fi Community Network**
  - An infinite population of heterogenous WiFi AP owners share their APs with each others;
  - Choose different sharing schemes, i.e., be Bills or Linus.



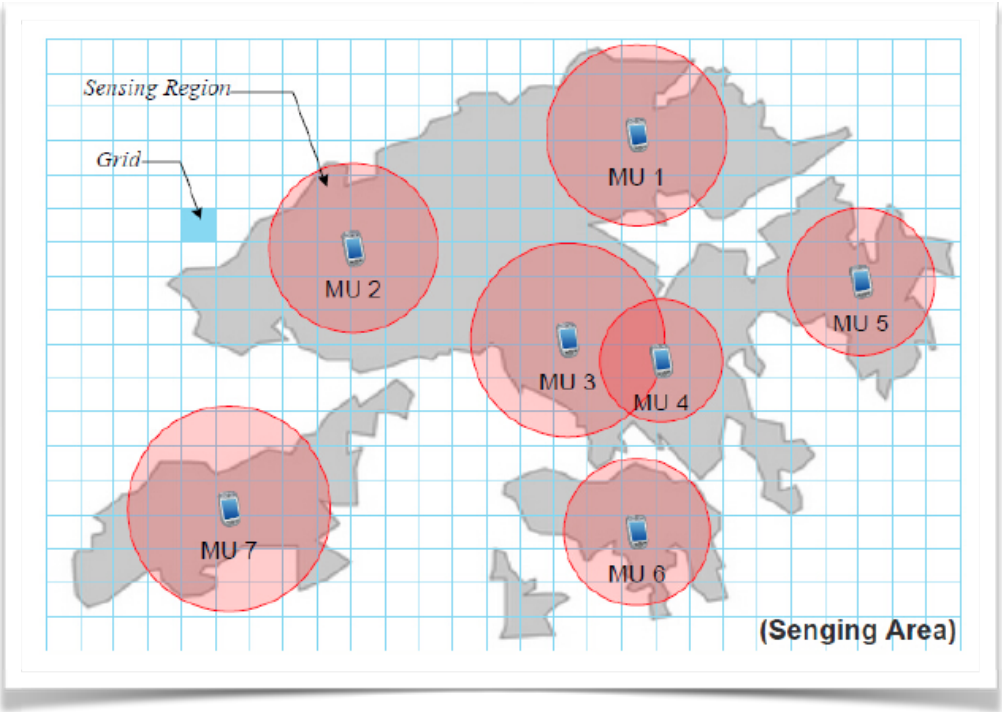
Application 3

- Wi-Fi Community Network
  - Users are heterogenous, with different data usage evaluations and roaming probabilities; (infinite and multi-dimensional type)
  - Population profile: (x, 1-x)
    - A population x of users choose to be Bills, and the remaining population 1-x choose to be Linus.
  - Payoff of each player:
    - (1) Payoff of a Bill increases with the population of Bills x;
    - (2) Payoff of a Linus is constant.

  - Key Problem: Equilibrium Analysis
    - What is the equilibrium population?
    - Is the equilibrium stable?

Application 4

- **Peer-to-Peer Mobile Crowd Sensing**
  - An infinite population of heterogenous mobile users sense and share data with each other;
  - Choose to sense data (and sell to others), or purchase data from sensing users.



Application 4

- Peer-to-Peer Mobile Crowd Sensing
  - Users are heterogenous, with different data values and sensing costs; (infinite and multi-dimensional type)
  - Population profile: (x, 1-x)
    - A population x of users choose to sense, and the remaining population 1-x choose to purchase data from sensing users;
  - Payoff of each player:
    - (1) Payoff of a sensing user decreases with the population x (sensing), and increases with the population 1-x (purchasing);
    - (2) Payoff of a purchasing user is constant.
  - Key Problem: Equilibrium Analysis
    - What is the equilibrium population?
    - Is the equilibrium stable?

Summary

- Introduce Population Game Theory;
- Discuss Real World Examples;
- Discuss Our Applications;

Thank You