ContrAuction: An Integrated Contract and Auction Design for Dynamic Spectrum Sharing

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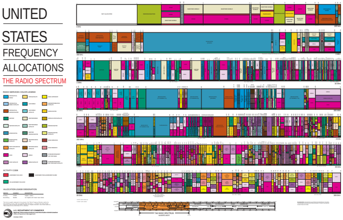
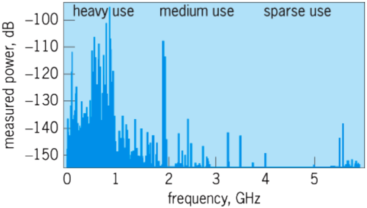
Background



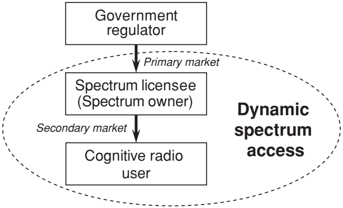
Frequency allocation (USA): *Over-crowed*



Spectrum utilization (Berkeley): *Inefficient*

- Dynamic Spectrum Access
  - Goal: Increase spectrum efficiency and alleviate spectrum scarcity
  - Basic idea: Allowing unlicensed users to access licensed spectrum
  - Requirement: Provide *economic incentive* for both primary spectrum owners and cognitive radio users ⇒ *Secondary Spectrum Market*

Secondary Spectrum Market



Spectrum primary market vs secondary market [E. Hossain, D. Niyato, Z. Han '09]

- Secondary Spectrum Market
  - Seller: Primary spectrum owner (PO)
  - Buyer: Cognitive radio user (secondary user, SU)
  - Item: Spectrums licensed to the PO
  - Scheme: Short-term vs Long-term
    - ★ e.g., in a slot-by-slot manner (millisecond or second scale)
- We focus on a *monopoly* secondary spectrum market (1 PO).

Hybrid Market Structure

- *Hybrid* Market Structure
  - Spot Market
    - Buyers compete openly for spectrums in a *real-time* and *on-demand* manner (e.g., through an *auction*)
    - Flexibility: Allow SUs to compete for spectrums based on their real-time demands ⇒ *Burst traffic* or *Elastic* services (e.g., file transferring)
  - Future Market
    - Buyers enter into certain *aforehand agreement* (called a *contract*, specifying the spectrum demand, price, etc.) with the seller
    - Certainty: Insures SUs (PO) against future uncertainty in market supply (demand) ⇒ *Period traffic* or *Inelastic* services (e.g., Netflix video streaming)
- Main advantage
  - Flexible in achieving desirable QoS differentiations

Our Contribution

Problem: PO's Profit Maximization

- *How should a monopoly PO sell his spectrums among contract users and spot market users to maximize his overall profit?*

- Novelty and main contribution
  - New modeling and solution technique
    - The first work tackling secondary spectrum trading with the coexistence of future and spot markets
  - Multiple information scenarios
    - Studying the optimal selling mechanisms under both information symmetry and asymmetry

The Network Model

- Network Model
  - One primary spectrum owner (PO)
    - ★ Transmission protocol: *Slotted* (e.g., GSM, WCDMA, and LTE)
    - ★ Spectrum opportunity: *Idle spectrum* (unused by licensed holders)
    - ★ Sharing scheme: *Short-term*, i.e., in a slot-by-slot manner
  - Multiple secondary users (SUs)
    - ★ Unlicensed: Eager for spectrums
    - ★ Valuation: Benefit from using some spectrums
    - ★ Service type: *Elastic* and *Inelastic* ⇒ Spot and Future market
  - Idle Spectrum
    - ★ *Un-reservable* ⇒ Allocate in real-time
    - ★ *Dynamic* across time ⇒ Not know future information
    - ★ *Heterogeneous* among users ⇒ User valuation diversity
    - ★ Each spectrum can only be used by one SU at the same time (Spatial reuse is not considered in this work)

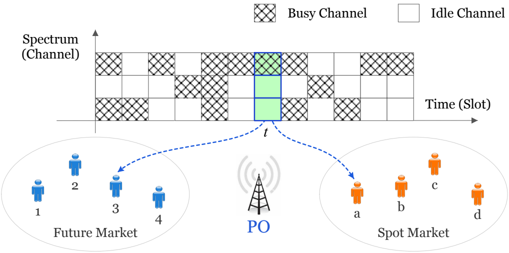
The Market Model

- Hybrid Spectrum Market Model
  - One seller (PO) – Monopoly market
    - ★ Trading scheme: *Short-term*, i.e., in a slot-by-slot manner
    - ★ $\mathcal{S}$: Idle spectrums in a specific time period (say T slots)
  - $\mathcal{M}$: Spot purchasing buyers (SUs) in the spot market
    - ★ Compete *openly* for spectrums only when needed
    - ★ Price based on the real-time valuation and market competition
    - ★ v_m: *Valuation* of spot market user $m \Rightarrow$ Maximal willingness-to-pay
  - $\mathcal{N}$: Contract buyers (SUs) in the future market
    - ★ Pre-defined spectrum *demand*, *payment* and *penalty* (in one period)
    - ★ $\mathbb{C}_n \triangleq \{B_n, D_n, \widehat{P}_n\}$: Contract signed by user $n \in \mathcal{N}$
    - ★ u_n: *Valuation* of contract user $n \Rightarrow$ Long-term satisfaction

The Network Information

- Network Information
  - All SUs' *valuations* for any idle spectrum
    - ⋆ Denoted by $\theta \triangleq (v_1, ..., v_M, u_1, ..., u_N)$ – *Random Vector*
  - Complete network information – Not practical !
    - ⋆ The PO knows the information θ of every spectrum *in advance*
  - Incomplete network information – $\surd$
    - ⋆ The PO does not know the precise information θ of every *future* spectrum, but only the *stochastic* distribution of θ, i.e., $f(\theta)$
    - ⋆ *Symmetric*: The PO can observe the SUs' realized valuations for the current spectrum (*but not those for the future spectrums*)
    - ⋆ *Asymmetric*: The PO *cannot* observe the SUs' realized valuations for the current spectrum
    - ⋆ Information Symmetry/Asymmetry $\Rightarrow$ Spot Trading Mechanism

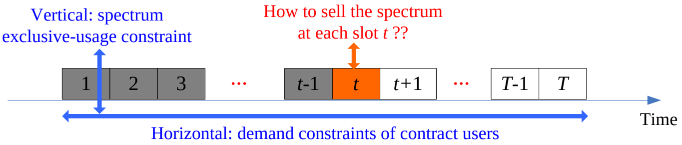
The Model – An example



Example 1: hybrid spectrum market [L. Gao, J. Huang, etc. 12']

- An example
  - Seller's supply: $\mathcal{S} = \{c_1, ..., c_{18}\}$ in total $T = 12$ slots
  - Spot market: $\mathcal{M} = \{$a, b, c, d$\}$
  - Future market: $\mathcal{N} = \{1, 2, 3, 4\}$
  - Two idle spectrums at time $T = t$ are sold to contract buyer 3 and spot market buyer a, respectively.

Problem Description and Approach



How to determine the allocation and charge for every spectrum in real-time under incomplete information?

- Approach – *Infinite-Dimensional Optimization*
  - Every information realization $\theta \Rightarrow$ Allocation strategy $\mathbf{A}(\theta)$

$$\mathbf{A}(\theta) \triangleq \big(a_0(\theta), a_1(\theta), ..., a_N(\theta)\big), \ \forall \theta \in \mathbf{\Theta}$$

    - ★ $a_0(\theta) \in [0, 1]$: allocation probability to the spot market
    - ★ $a_n(\theta) \in [0, 1]$: allocation probability to the contract user $n \in \mathcal{N}$
  - Constraints
    - ★ *User-coupling* constraint: $\sum_{n=0}^{N} a_n(\theta) \leq 1, \forall \theta \in \mathbf{\Theta}$
    - ★ *Time-coupling* constraint: $\int_{\theta} a_n(\theta) f(\theta) \mathrm{d}\theta < \frac{D_n}{S} \Rightarrow$ *Penalty*, $\forall n \in \mathcal{N}$

Problem Formulation

Problem Formulation – PO's Profit Maximization

$$\underset{\mathbf{A}(\theta),\forall\theta}{\textbf{maximize}} \quad \mathbb{E}[R_0] + \sum_{n=1}^{N} \mathbb{E}[R_n] - \sum_{n=1}^{N} w_n \mathbb{E}[C_n],$$

$$\textbf{subject to} \quad a_n(\theta) \in [0,1],\ \forall n \in \{0,1,...,N\}, \forall\theta \in \mathbf{\Theta};$$

$$\textstyle\sum_{n=0}^{N} a_n(\theta) \le 1,\ \forall\theta \in \mathbf{\Theta}.$$

- $\mathbb{E}[R_0] \triangleq S \int_\theta a_0(\theta) r_0(\theta) f(\theta) \mathrm{d}\theta$: *Expected profit* from the spot market
  - $r_0(\theta)$: PO's maximum profit as selling a spectrum θ on the spot market, depending on spot trading mechanism
- $\mathbb{E}[R_n] \triangleq B_n - [D_n - \mathbb{E}[d_n]]^+ \widehat{P}_n$: *Expected profit* from the contract user n
- $\mathbb{E}[C_n] \triangleq S \int_\theta a_n(\theta) c_n(\theta) f(\theta) \mathrm{d}\theta$: *Expected cost* from the contract user n
  - w_n: weight of contract user n's long-term satisfaction loss (cost)
- $\mathbb{E}[d_n] \triangleq S \int_\theta a_n(\theta) f(\theta) \mathrm{d}\theta$: *Expected number* of spectrums to contract user n

Information Symmetry

- The PO can observe the SUs' realized valuations at each time slot
  - Optimal Pricing: Charging whatever the spot market will bear

Spot Trading Mechanism – Perfect Price Discrimination

$$r_0(\theta) = Y_M^1(\theta) \triangleq \max_{m \in \mathcal{M}} v_m$$

- Allocate each spectrum to the highest valuation user
- Charge the allocated user a price *exactly same* as its valuation

Equivalent Transform

- Condition I (*Full Spectrum Utilization*)
  - $\sum_{n=0}^{N} a_n^*(\theta) = 1 \Rightarrow a_0^*(\theta) = 1 - \sum_{n=1}^{N} a_n^*(\theta)$
- Condition II (*No Contract Overflow*)
  - $\mathbb{E}[d_n] \triangleq S \int_\theta a_n(\theta) f(\theta) \mathrm{d}\theta \leq D_n \Rightarrow \mathbb{E}[R_n] \triangleq B_n - (D_n - \mathbb{E}[d_n]) \cdot \widehat{P}_n$

Equivalent Optimization Problem

$$\underset{\mathbf{A}_0(\theta), \forall \theta}{\textbf{maximize}} \quad F + S \cdot \sum_{n=1}^{N} \int_\theta H_n(\theta) a_n(\theta) f(\theta) \mathrm{d}\theta$$

$$\textbf{subject to} \quad \text{(i)} \quad a_n(\theta) \geq 0, \ \forall n \in \mathcal{N}, \forall \theta \in \mathbf{\Theta}$$

$$\text{(ii)} \quad \sum_{n=1}^{N} a_n(\theta) \leq 1, \ \forall \theta \in \mathbf{\Theta}$$

$$\text{(iii)} \quad \mathbb{E}[d_n] \leq D_n, \ \forall n \in \mathcal{N}$$

- $\mathbf{A}_0(\theta) \triangleq \mathbf{A}(\theta)/\{a_0(\theta)\} = (a_1(\theta), ..., a_N(\theta))$: *New allocation strategy*
- $H_n(\theta) \triangleq -r_0(\theta) + \widehat{P}_n - w_n c_n(\theta)$
- $F \triangleq S \cdot \int_\theta r_0(\theta) f(\theta) \mathrm{d}\theta + \sum_{n=1}^{N} (B_n - \widehat{P}_n D_n)$: *Constant*

Primal-Dual Method

- Dual Variables: $\mu_n(\theta)$, $\eta(\theta)$, λ_n, $\forall n \in \mathcal{N}$
- Lagrangian:

$$\mathbb{L} \triangleq \int_\theta \mathcal{L}(\theta) f(\theta) \mathrm{d}\theta$$

Sub-Lagrangian – $\mathcal{L}(\theta)$

$$\mathcal{L}(\theta) \triangleq F + S \sum_{n=1}^{N} H_n(\theta) a_n(\theta) + \sum_{n=1}^{N} \mu_n(\theta) a_n(\theta) + \eta(\theta)\Big(1 - \sum_{n=1}^{N} a_n(\theta)\Big) + \sum_{n=1}^{N} \lambda_n \big(D_n - S \cdot a_n(\theta)\big)$$

First-order Derivative of $\mathcal{L}(\theta)$

Marginal Profit

$$\mathcal{L}^{(n)}(\theta) \triangleq \frac{\partial \mathcal{L}(\theta)}{\partial a_n(\theta)} = S \cdot H_n(\theta) + \mu_n(\theta) - \eta(\theta) - S \cdot \lambda_n$$

Mantle Marginal Profit

$$\mathcal{J}_1^{(n)}(\theta) \triangleq S \cdot H_n(\theta) - \eta(\theta) - S \cdot \lambda_n$$

Core Marginal Profit

$$\mathcal{J}_2^{(n)}(\theta) \triangleq S \cdot H_n(\theta) - S \cdot \lambda_n$$

- $\mathcal{L}^{(n)}(\theta)$: *marginal profit*, the *first-order derivative* of $\mathcal{L}(\theta)$ with respect to $a_n(\theta)$
- $\mathcal{J}_1^{(n)}(\theta) = \mathcal{L}^{(n)}(\theta) - \mu_n(\theta)$: eliminate $\mu_n(\theta)$ from the *marginal profit*
- $\mathcal{J}_2^{(n)}(\theta) = \mathcal{L}^{(n)}(\theta) - \mu_n(\theta) + \eta(\theta)$: eliminate $\mu_n(\theta)$ and $\eta(\theta)$ from the *marginal profit*

First-order Condition and Duality Principle

Optimal Primary Solution – $a_n^*(\theta)$

$$a_n^*(\theta) = \begin{cases} 0, & \mathcal{L}^{(n)}(\theta) < 0 \\ 1, & \mathcal{L}^{(n)}(\theta) > 0 \\ \delta \in [0,1], & \mathcal{L}^{(n)}(\theta) = 0 \end{cases}$$

Dual Constraints

- $\mu_n(\theta) \geq 0, \quad a_n^*(\theta) \geq 0, \quad \mu_n(\theta) a_n^*(\theta) = 0, \quad \forall n \in \mathcal{N}, \theta \in \mathbf{\Theta}$
- $\eta(\theta) \geq 0, \quad 1 - \sum_{n=1}^{N} a_n^*(\theta) \geq 0, \quad \eta(\theta)\big(1 - \sum_{n=1}^{N} a_n^*(\theta)\big) = 0, \quad \forall \theta \in \mathbf{\Theta}$
- $\lambda_n \geq 0, \quad D_n - \mathbb{E}[d_n] \geq 0, \quad \lambda_n \big(D_n - \mathbb{E}[d_n]\big) = 0, \quad \forall n \in \mathcal{N}$

- Duality Principle
  - *Finding optimal primary solution* $\Leftrightarrow$ *Finding optimal dual variables satisfying dual constraints*

Optimal Dual Variables – $\mu_n^*(\theta)$, $\eta^*(\theta)$

Lemma 3 – Optimal conditions for $\mu_n^*(\theta)$

$$\begin{cases} \mathcal{J}_1^{(n)}(\theta) \geq 0 \Rightarrow \mu_n^*(\theta) = 0 \\ \mathcal{J}_1^{(n)}(\theta) < 0 \Rightarrow \mu_n^*(\theta) \in [0,\ |\mathcal{J}_1^{(n)}(\theta)|] \end{cases}$$

- $\mu_n^*(\theta)$ never changes the sign of marginal profit: $\text{sign}\{\mathcal{L}^{(n)}(\theta)\} \equiv \text{sign}\{\mathcal{J}_1^{(n)}(\theta)\}$

Lemma 4 – Optimal conditions for $\eta^*(\theta)$

$$\begin{cases} K_1(\theta) \geq 0 \Rightarrow \eta^*(\theta) \in [\max(0, K_2(\theta)),\quad K_1(\theta)] \\ K_1(\theta) < 0 \Rightarrow \eta^*(\theta) = 0 \end{cases}$$

- $K_1(\theta) \triangleq \max_{n\in\mathcal{N}} \mathcal{J}_2^{(n)}(\theta)$: the highest core marginal profit
- $K_2(\theta) \triangleq \max_{n\in\mathcal{N}/n_1} \mathcal{J}_2^{(n)}(\theta)$: the second highest core marginal profit
- $\eta^*(\theta)$ reduces identically all marginal profits such that at most one is positive

Optimal Dual Variables – λ_n^*

Lemma 6 – Optimal conditions for λ_n^*

$$\lambda_n^* = \max\left\{0, \quad \arg_{\lambda_n} S \cdot \int_{\theta \in \boldsymbol{\Theta}_n^+(\boldsymbol{\Lambda}_{-n}^*, \lambda_n)} f(\theta)\mathrm{d}\theta = D_n\right\}$$

- $\boldsymbol{\Theta}_n^+ \triangleq \{\theta | \mathcal{J}_2^{(n)}(\theta) > 0 \& \mathcal{J}_2^{(n)}(\theta) > \max_{i \neq n} \mathcal{J}_2^{(i)}(\theta)\}$: spectrums to contract user n.
- λ_n^* shifts vertically user n's marginal profit to meet demand constraint $\mathbb{E}[d_n] \leq D_n$

Optimal solution – $a_n^*(\theta)$

$$a_n^*(\theta) = 1 \Leftrightarrow \mathcal{J}_2^{(n)}(\theta) \geq 0 \ \& \ \mathcal{J}_2^{(n)}(\theta) \geq \max_{i \neq n} \mathcal{J}_2^{(i)}(\theta)$$

- Intuitively, allocate each spectrum with θ to the contract with *highest* and *positive* core marginal profit $\mathcal{J}_2^{(n)}(\theta)$

Solution Summary (Information Symmetry)

Optimal Selling Mechanism – Perfect Price Discrimination

- *Price definition*
  - $p_m(\theta) \triangleq v_m$ for spot market user $m \in \mathcal{M}$
  - $p_n(\theta) \triangleq \widehat{P}_n - w_n c_n(\theta) - \lambda_n^*$ for contract user $n \in \mathcal{N}$
- *Allocation strategy*
  - Allocate each spectrum θ to the *highest price* user
- *Charge scheme*
  - Charge user's price or valuation v_m if a spot market user m wins
  - Charge a pre-defined price if a contract user n wins

- Comments
  - Contract user's price depends on *penalty* rather than payment;
  - Shadow price λ_n^* reduces contract user n's price to meet demand constraint;

Information Asymmetry

- The PO *cannot* observe the SUs' realized valuations at each time slot
  - Incentive compatible mechanism ⇒ Truth-telling of spot market users

Spot Trading Mechanism – VCG-based Auction

$$r_0(\theta) = Y_M^2(\theta) \triangleq \max_{m \neq m^*} v_m$$

- Allocate each spectrum to the highest bid user
- Charge the allocated user a *critical value* (e.g., the second highest bid in a second-price auction)

Auction in Hybrid Market

- Challenge I
  - How to involve contract users into the spot auction?
    - *Not willing to* be involved in the competition of an auction
    - *Not able to* be involved in the competition of an auction
  - Solution – ContrAuction
    - The PO acts as *virtual bidders* on behalf of the contracts
    - Mechanism is *transparent* to the contract users
- Challenge II
  - How to determine the optimal bid for each contract?
    - *No external relevance*: each contract user's bid is irrelevant to other users' information ⇒ ensure *truthfulness* of spot market users
    - *Efficiency constraint*: achieve the same spectrum allocation as in information symmetry ⇒ outcome is *transparent* to the contract users

ContrAuction and Optimal Bidding Rule

Integrated Contract and Auction Design – ContrAuction

- An *VCG-based Auction* as the underlying spot trading mechanism
- Basic idea: the PO acts as *virtual bidders* on behalf of the contracts

  - Mechanism is *truthful* to the spot market users
  - Mechanism and Outcome are *transparent* to the contract users

Optimal Bidding Rule (under Efficiency Constraint)

$$b_n^*(\theta) \triangleq \widehat{P}_n - w_n c_n(\theta) - \lambda_n^*$$

  - $b_n^*(\theta)$: contract user n's own information and a shadow price λ_n^* given by Lemma 6
  - Efficiency: achieves the *same allocation* as in information symmetry
  - Optimality: maximizes the PO's profit among all *efficient* mechanisms.

Solution Summary (Information Asymmetry)

Optimal Selling Mechanism – ContrAuction

- *Bidding strategy*
  - $b_m(\theta) \triangleq v_m$ for spot market user $m \in \mathcal{M}$ (*truthfulness*)
  - $b_n(\theta) \triangleq \widehat{P}_n - w_n c_n(\theta) - \lambda_n^*$ for contract user $n \in \mathcal{N}$
- *Allocation strategy*
  - Allocate each spectrum θ to the *highest bid* user
- *Charge scheme*
  - Charge the *second highest bid* if a spot market user wins
  - Charge a pre-defined price if a contract user wins

- Comments
  - Contract user's bid is same as the "price" in information symmetry;
  - Contract user's bid has exactly the same effect as a reserve price.

Conclusion and Future Work

- Conclusion
  - *Secondary spectrum trading* with the coexistence of future and spot markets;
  - *PO's profit maximization* under incomplete information;
  - *Optimal selling mechanisms* under both information symmetry and asymmetry.
- Future Work
  - *Spatial Reuse*: interference protocol model and physical model
  - *Without efficiency constraint*: optimal ContrAuction mechanism

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